

PUNOSTIJENI GLAVNI NOSAČI

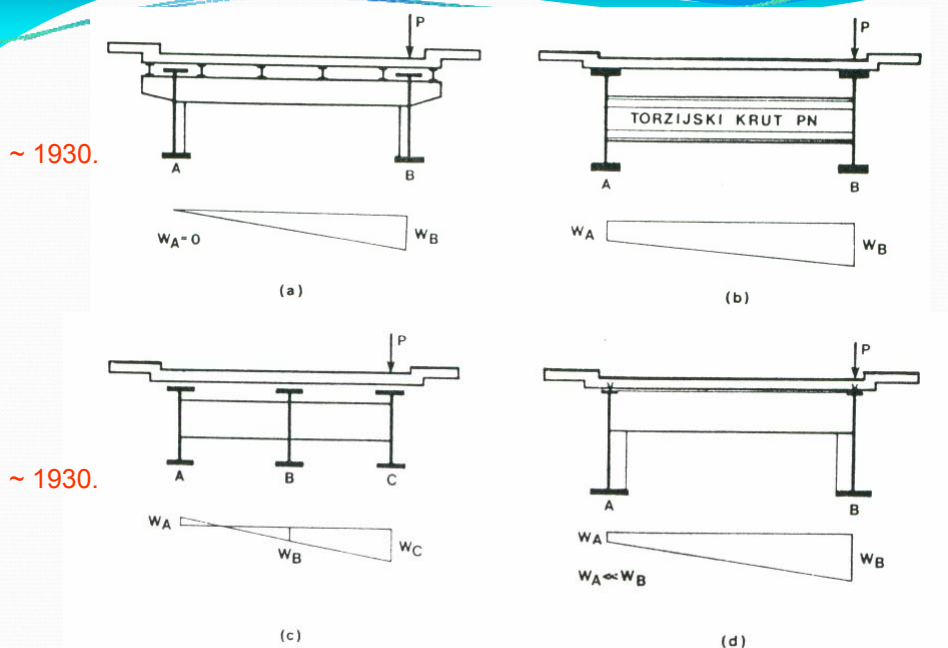
Poslije "Britannia" mosta tek 30-tih godina 20. stoljeća.

1. Uskopojasni + kombinacija roštilji, torzija, sprezanje
2. Širokopojasni:
 - otvoreni presjeci
 - zatvoreni presjeci

Izvedba:

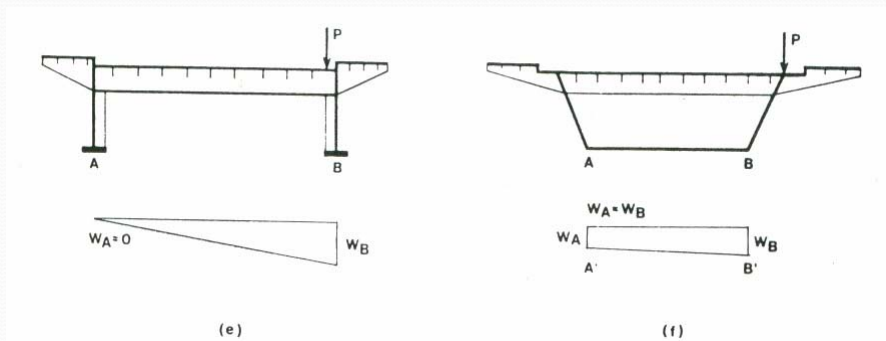
- Zakovani
- Zavareni
- Kombinirano
 - zakovice, zavarivanje
 - vijci, zavarivanje

RAZVOJ PUNOSTIJENIH NOSAČA



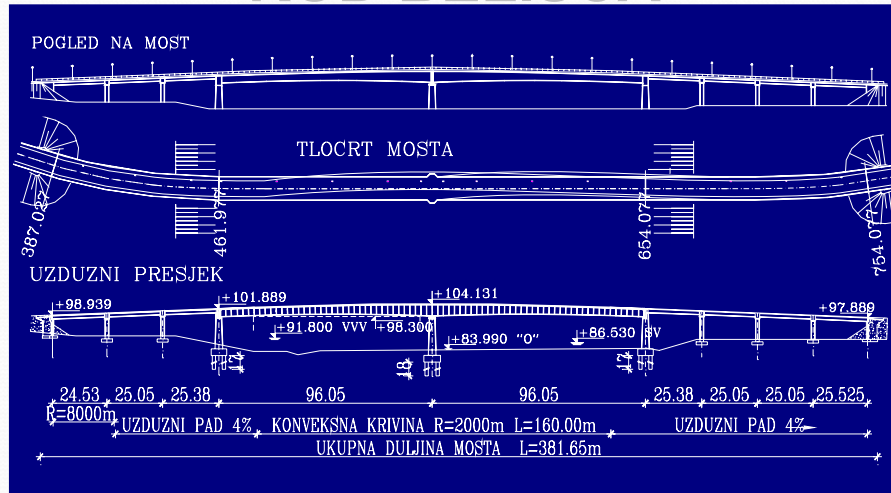
RAZVOJ PUNOSTIJNIH NOSAČA

~ 1950. do danas

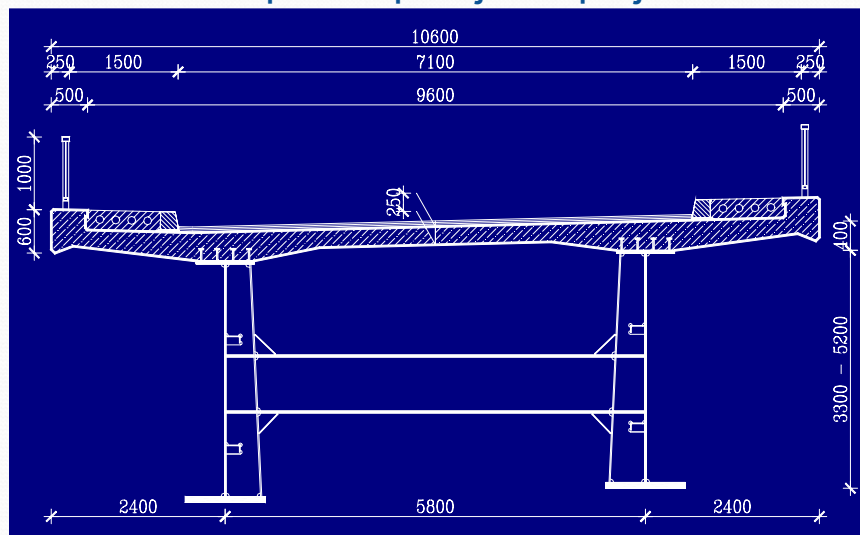


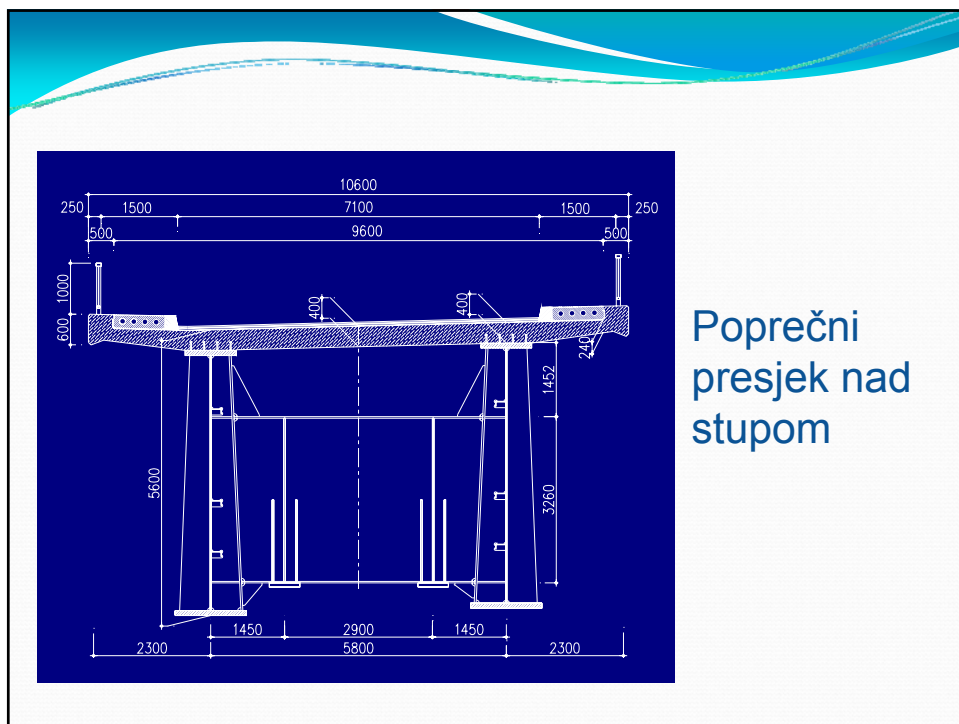
GREDNI MOSTOVI

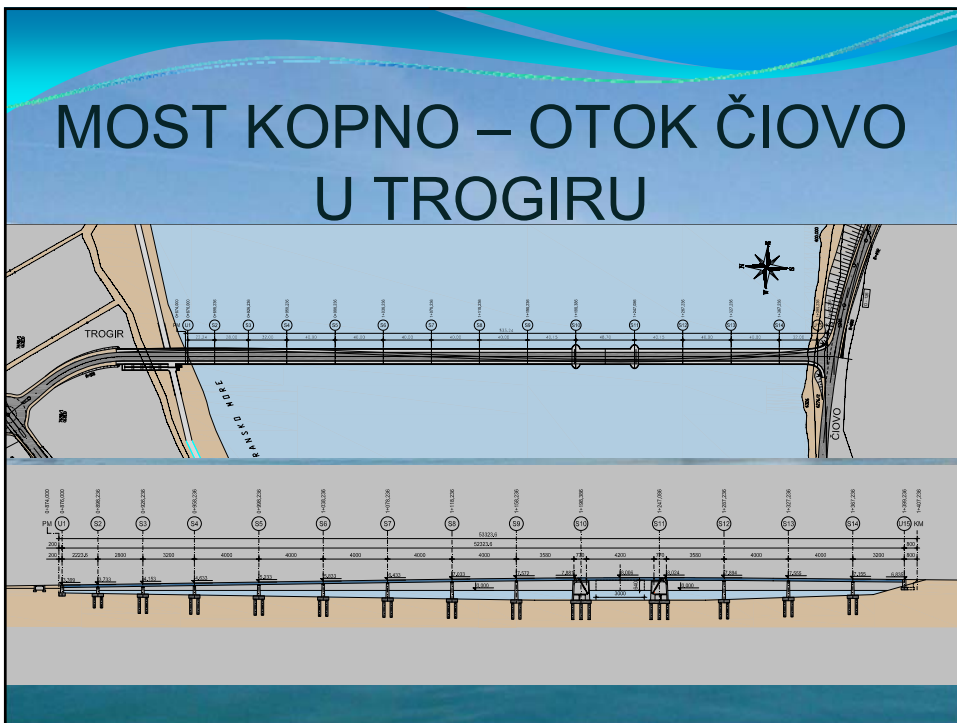
MOST PREKO RIJEKE DRAVE KOD BELIŠĆA

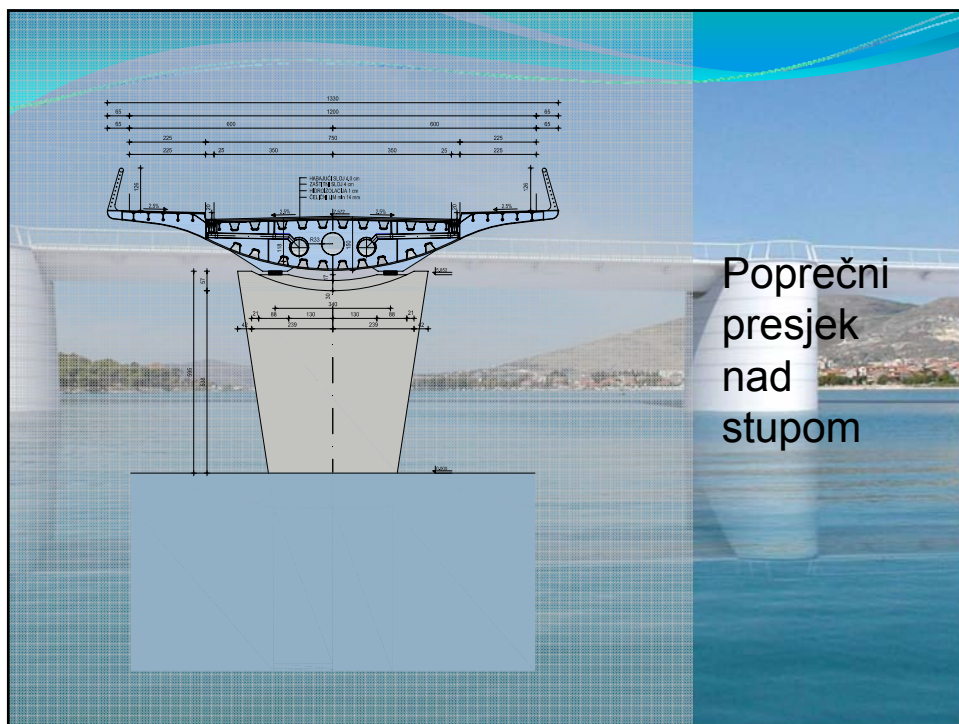


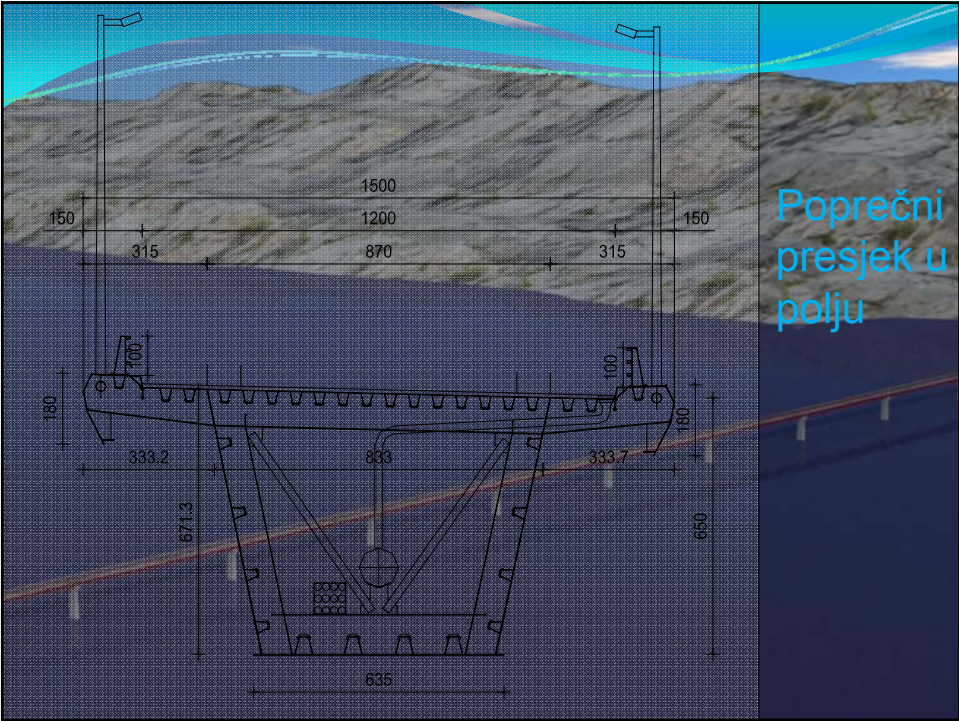
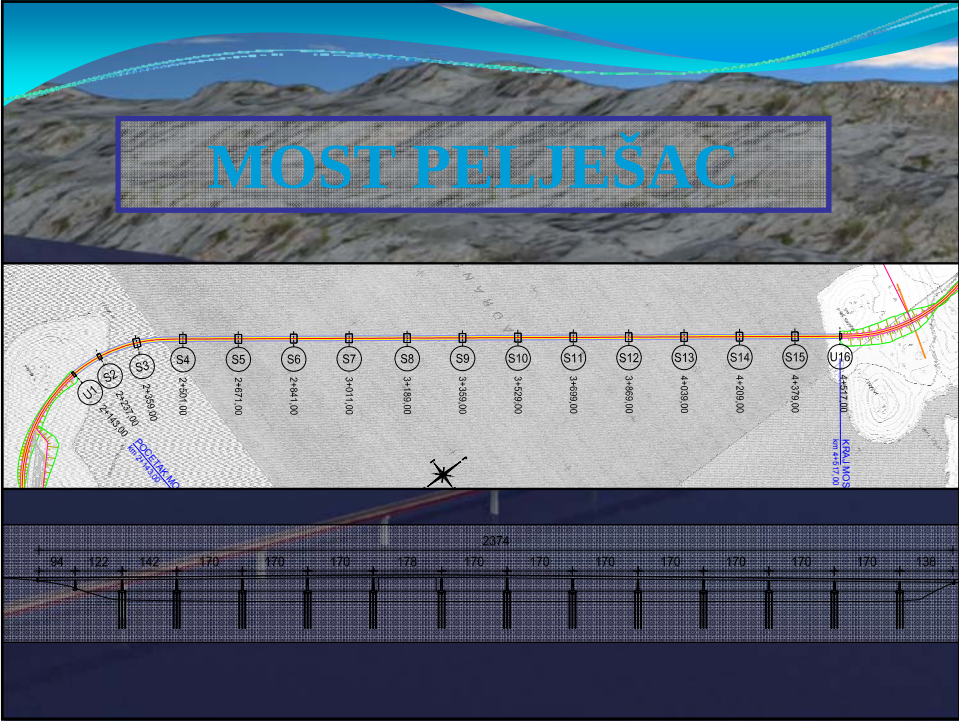
Poprečni presjek u polju

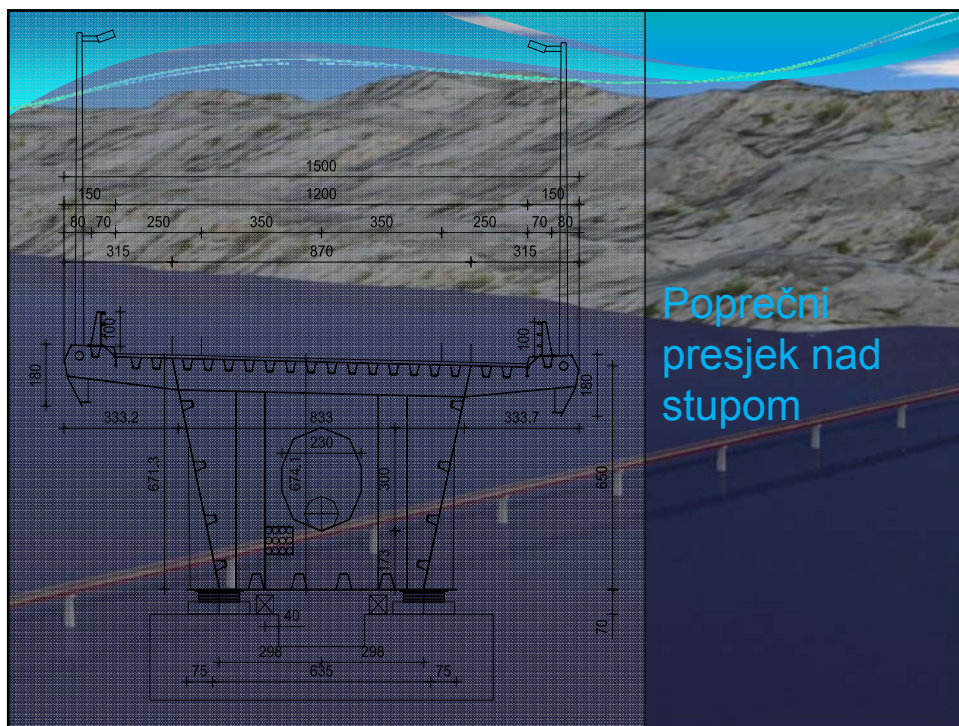




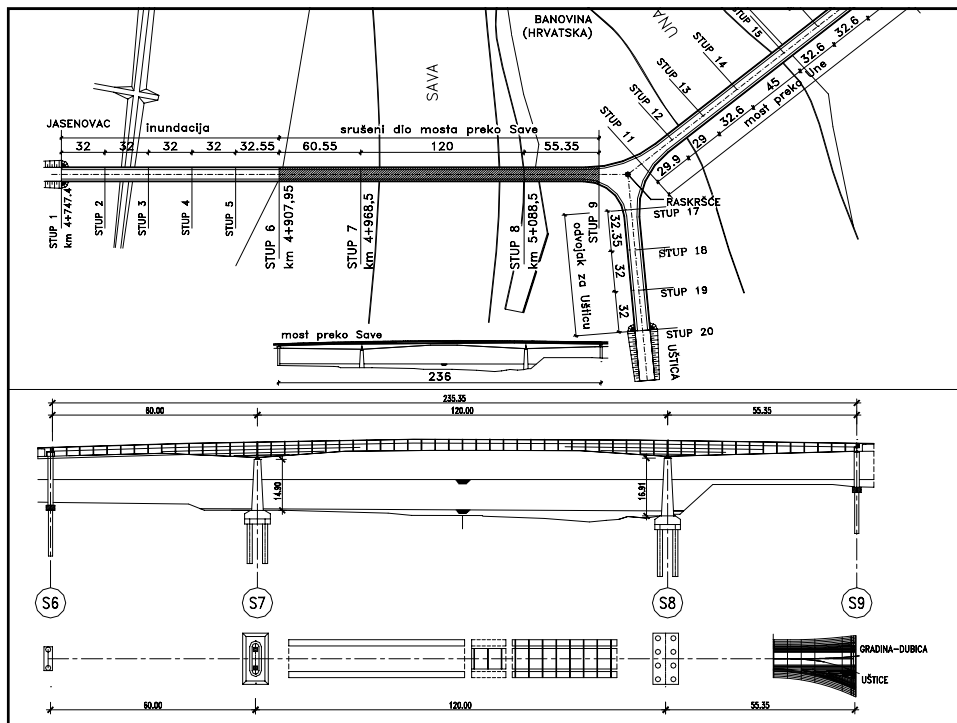


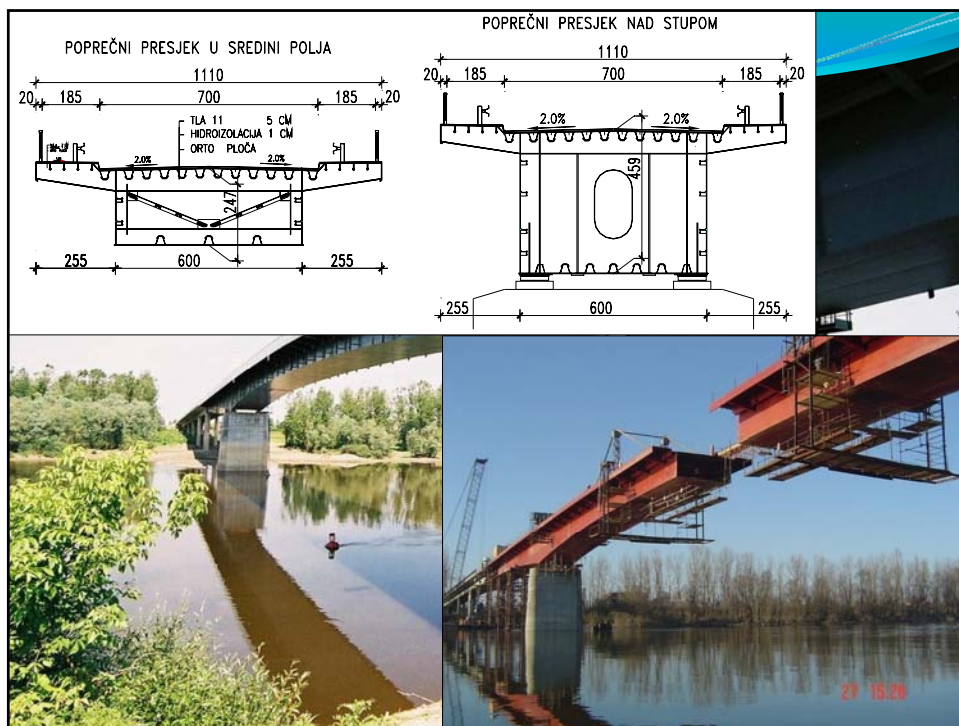


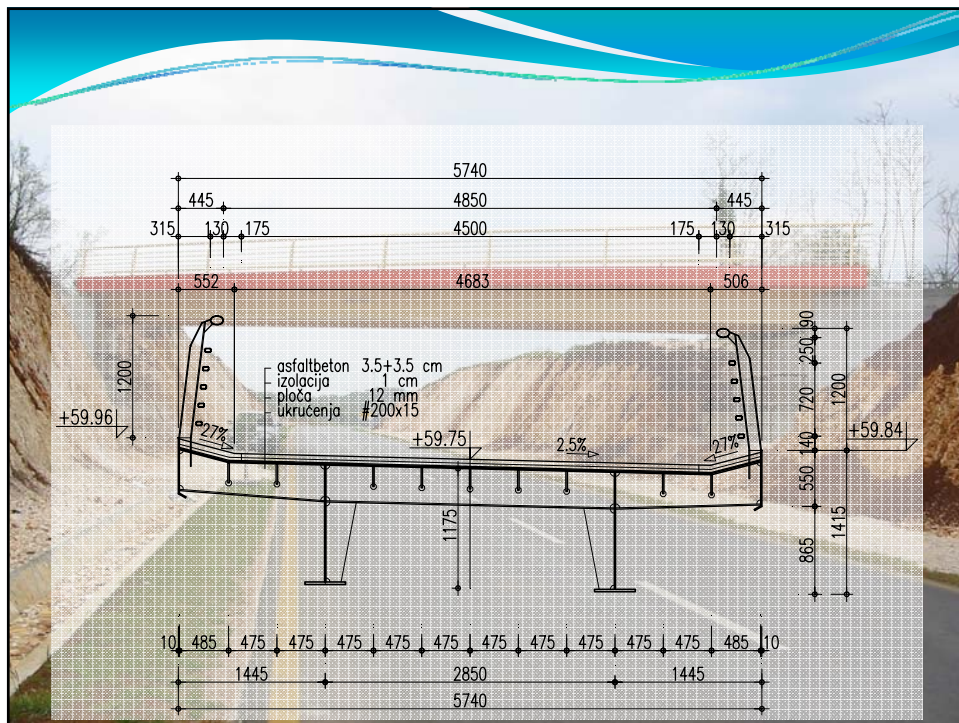
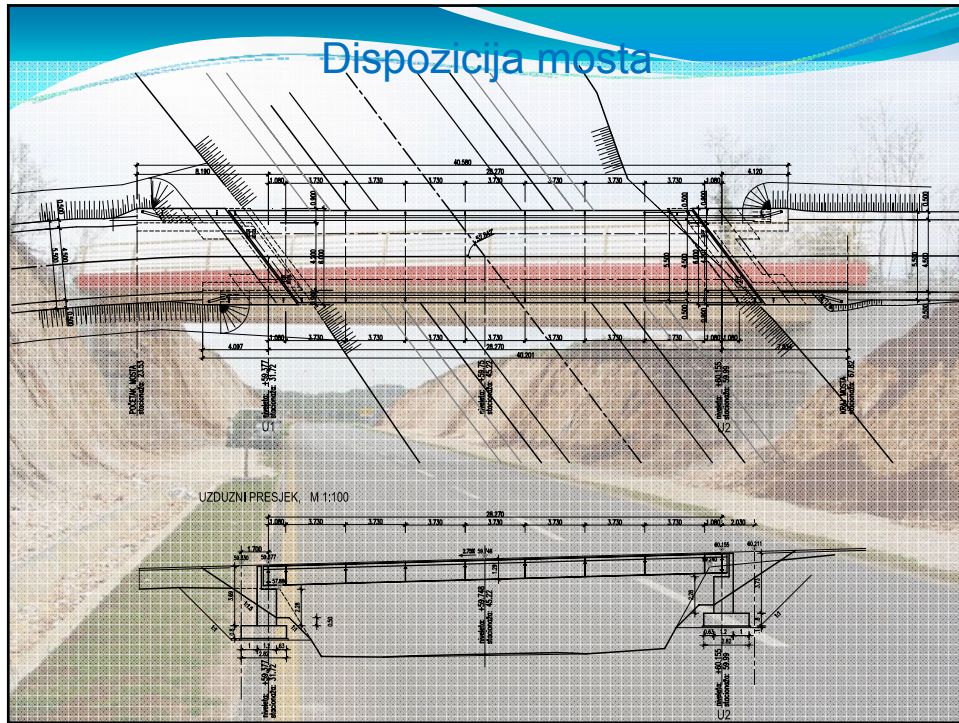




MOST PREKO SAVE I UNE U JASENOVCU

A large concrete bridge with multiple spans crosses a wide river. The bridge has a high, flat deck and concrete piers. A person is swimming in the river below the bridge. The river is surrounded by lush green trees and vegetation. The sky is blue with some clouds. The text 'MOST PREKO SAVE I UNE U JASENOVCU' is overlaid on the top left of the image.

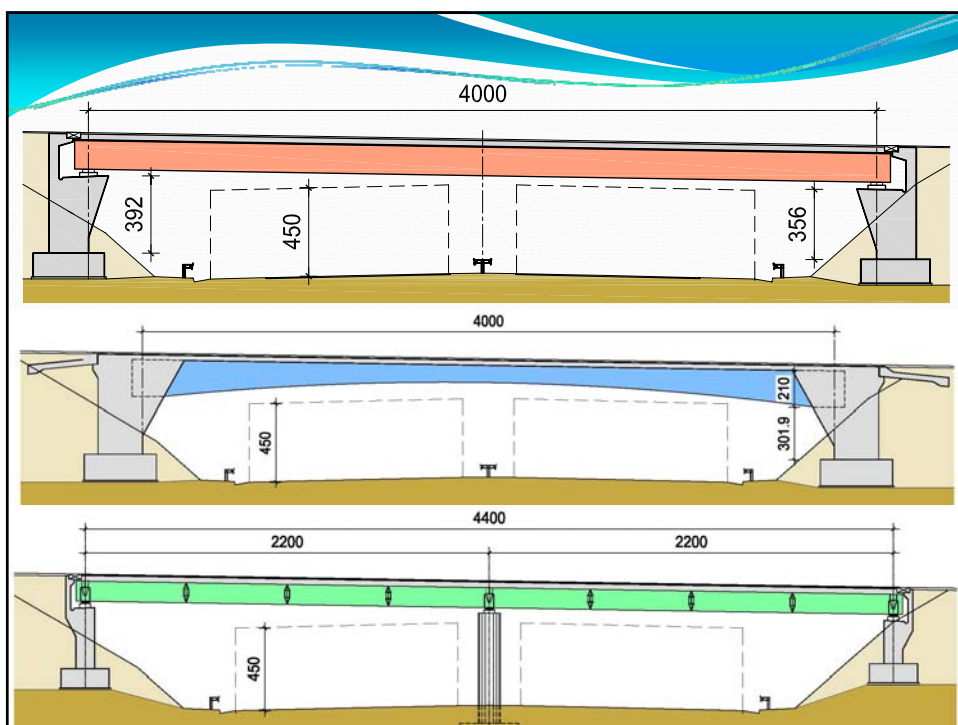


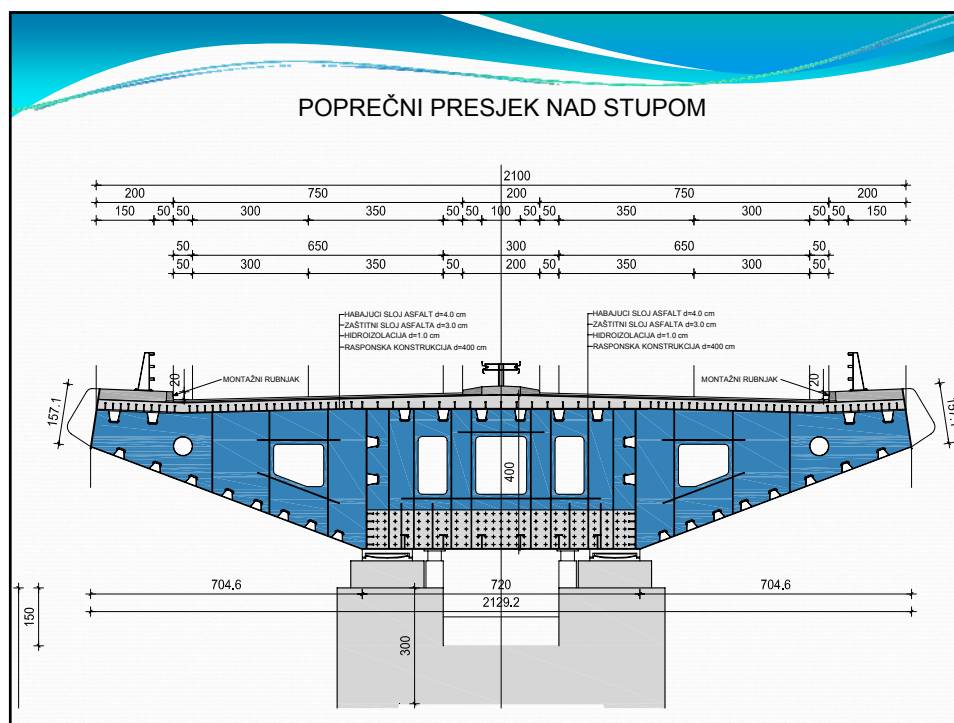




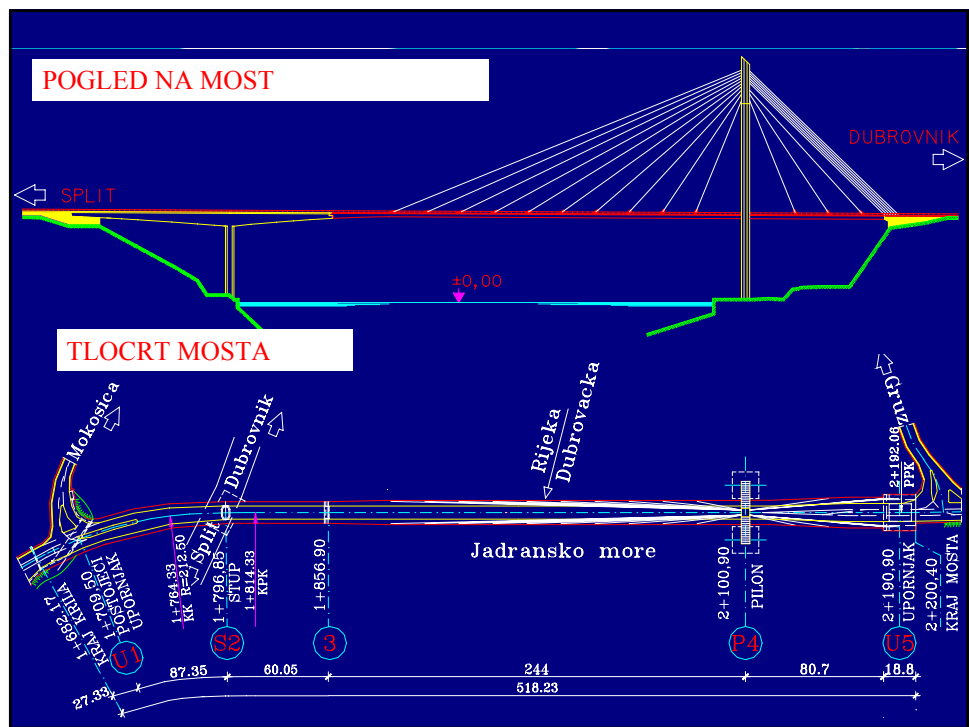


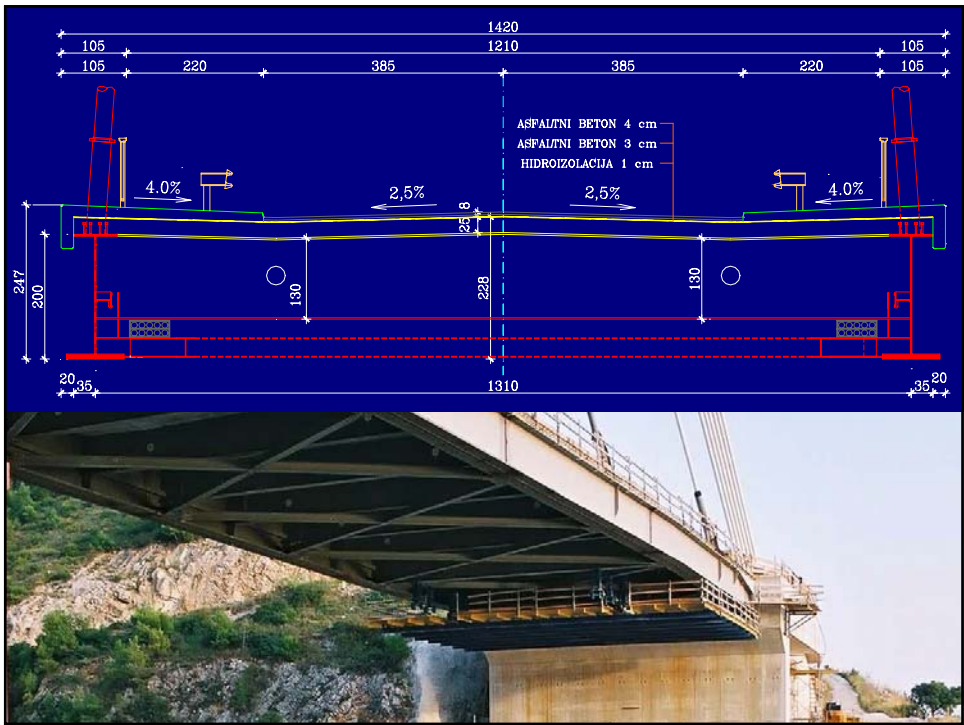
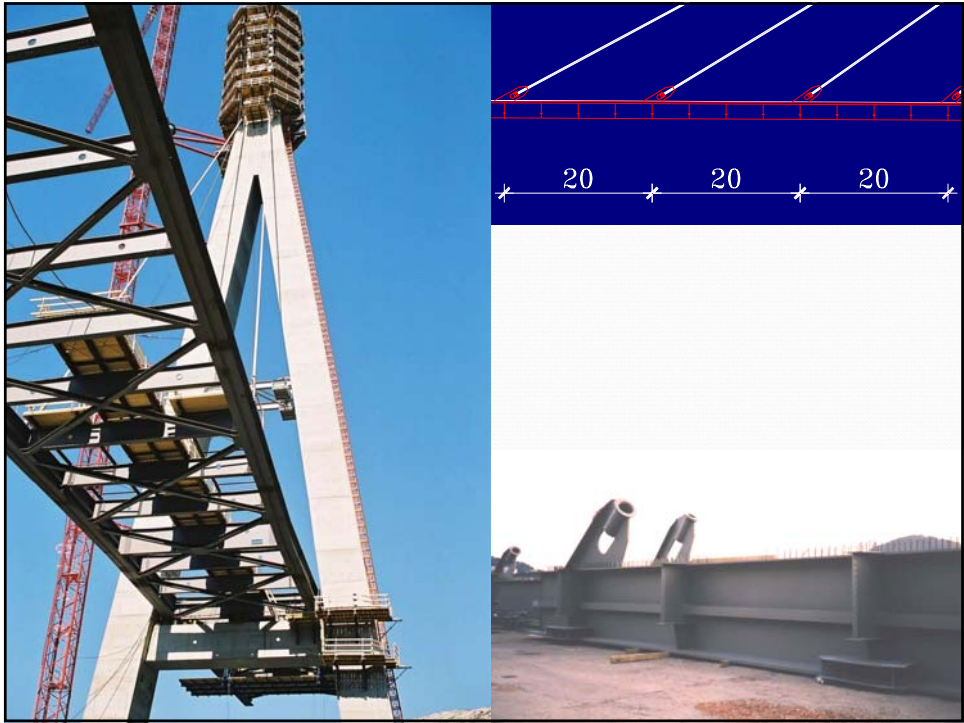
NADVOŽNJAK MUČÍČI



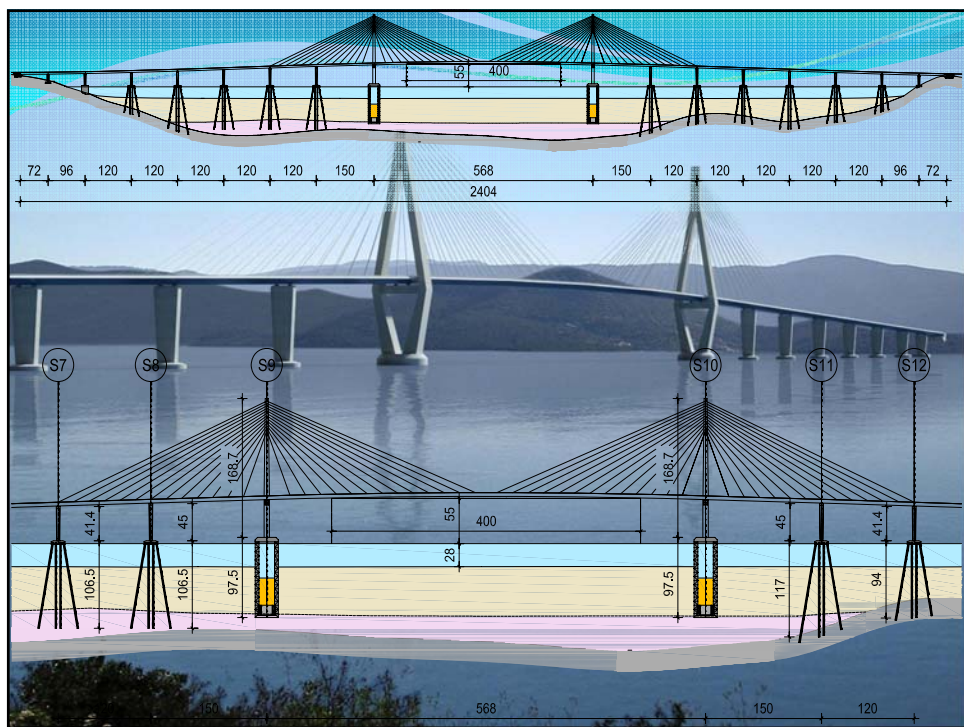
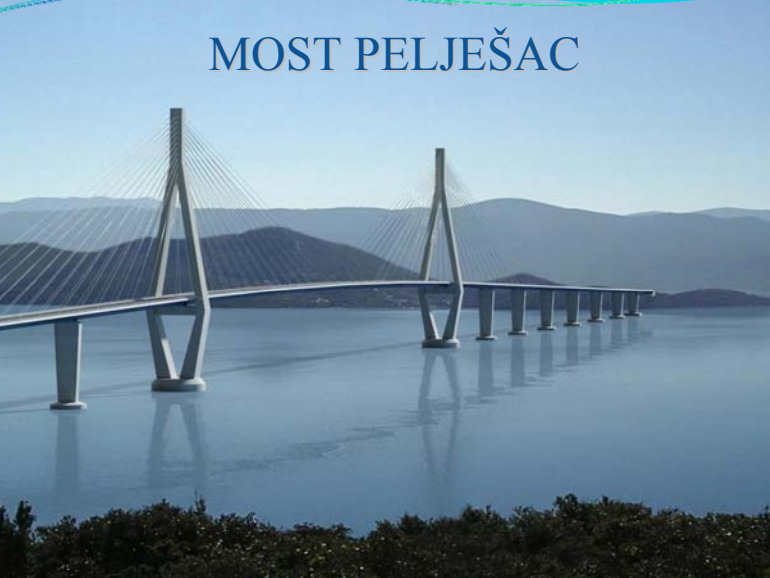


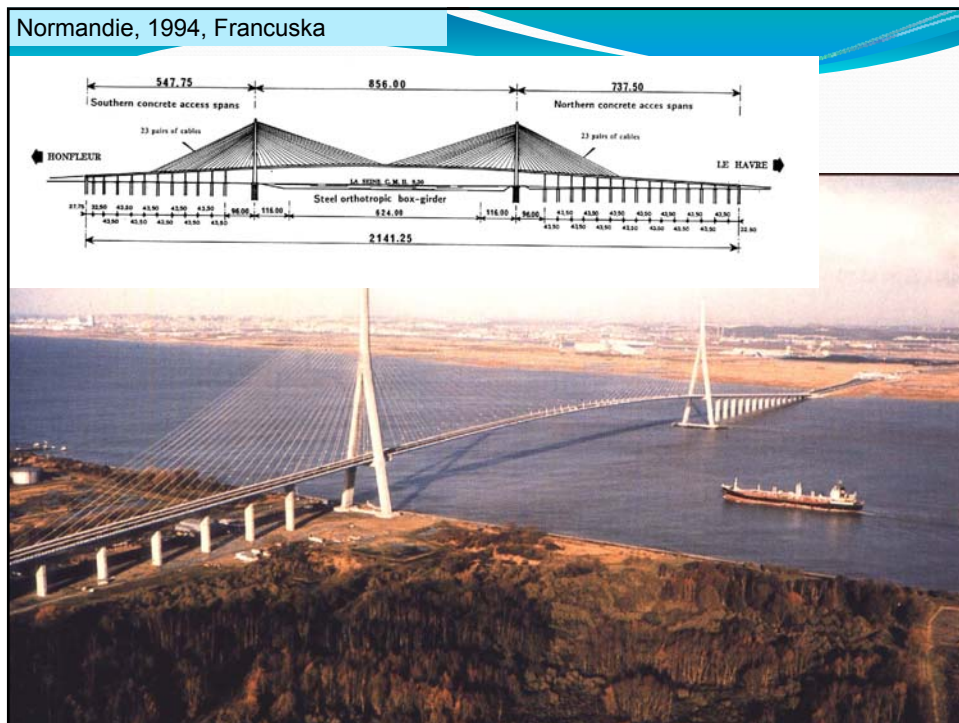
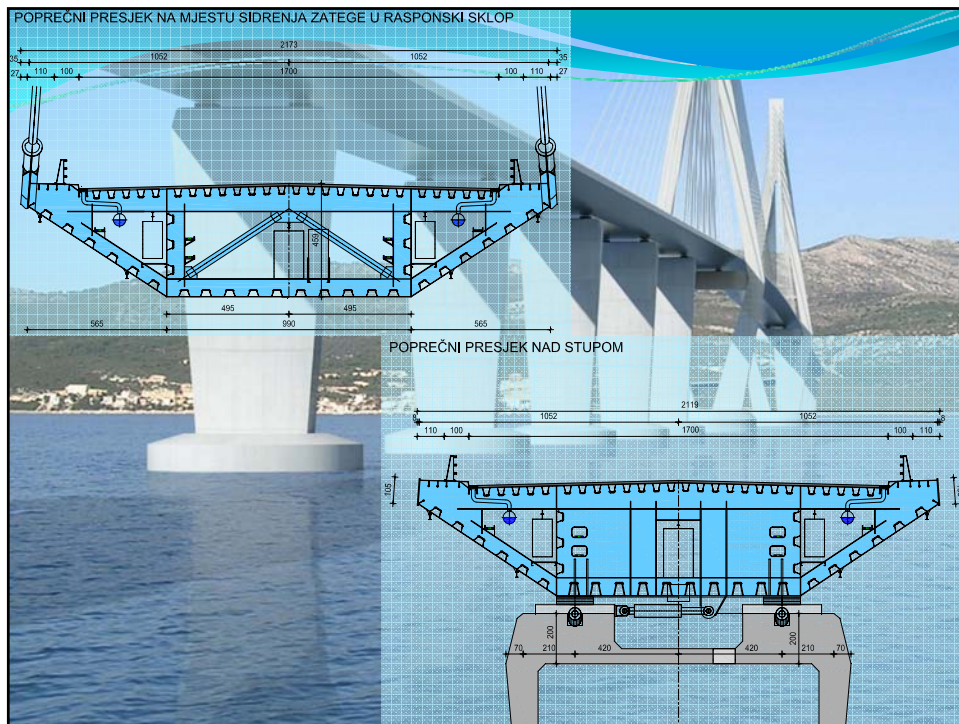
ZAVJEŠENI MOSTOVI

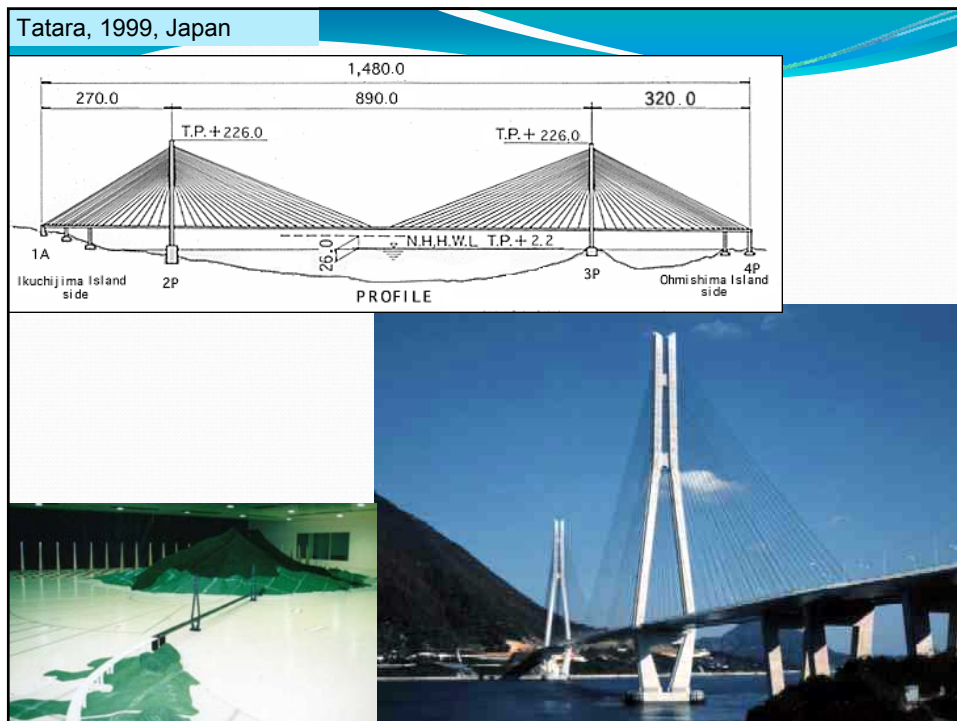
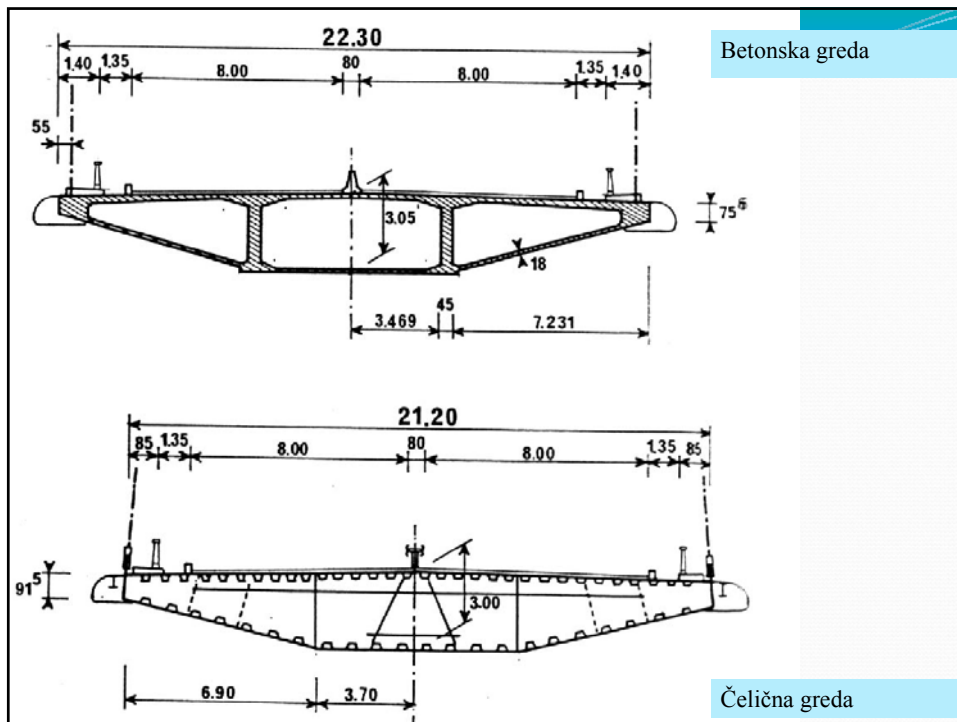


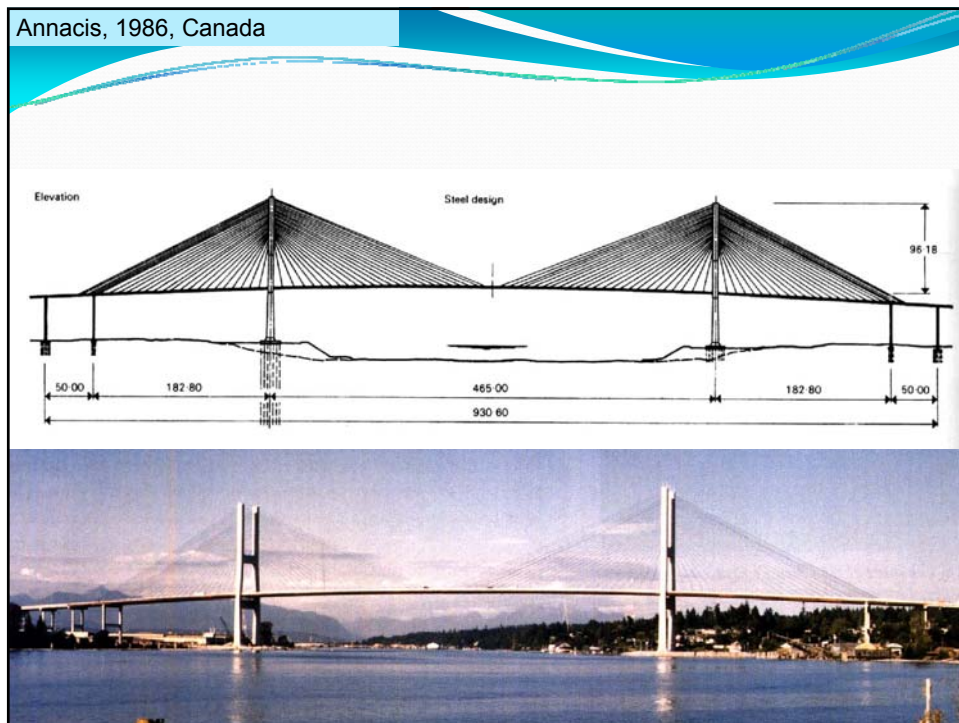
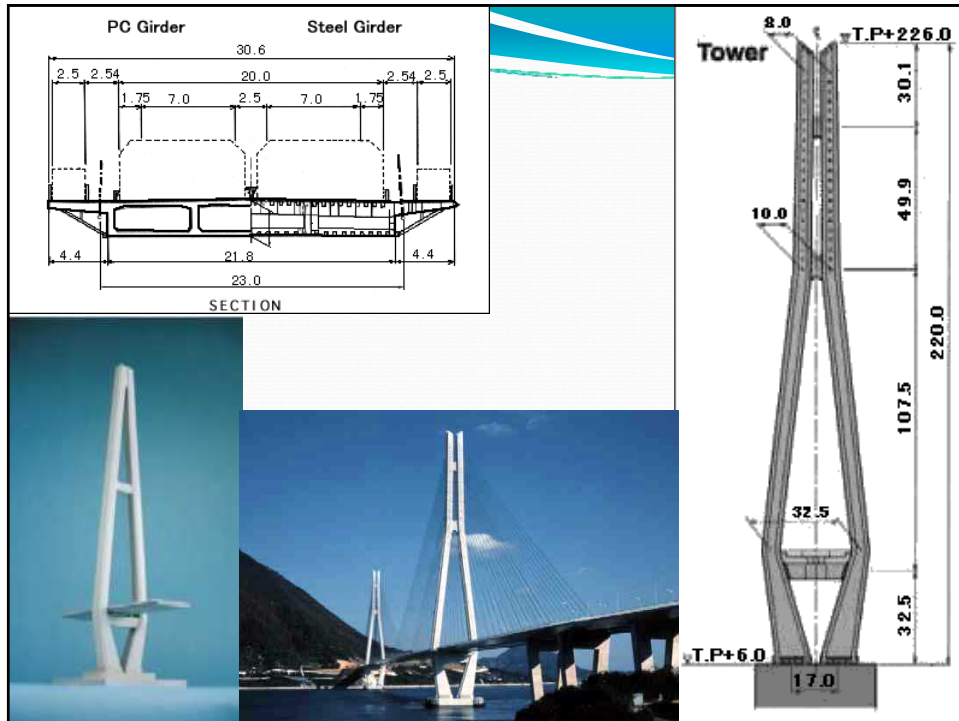


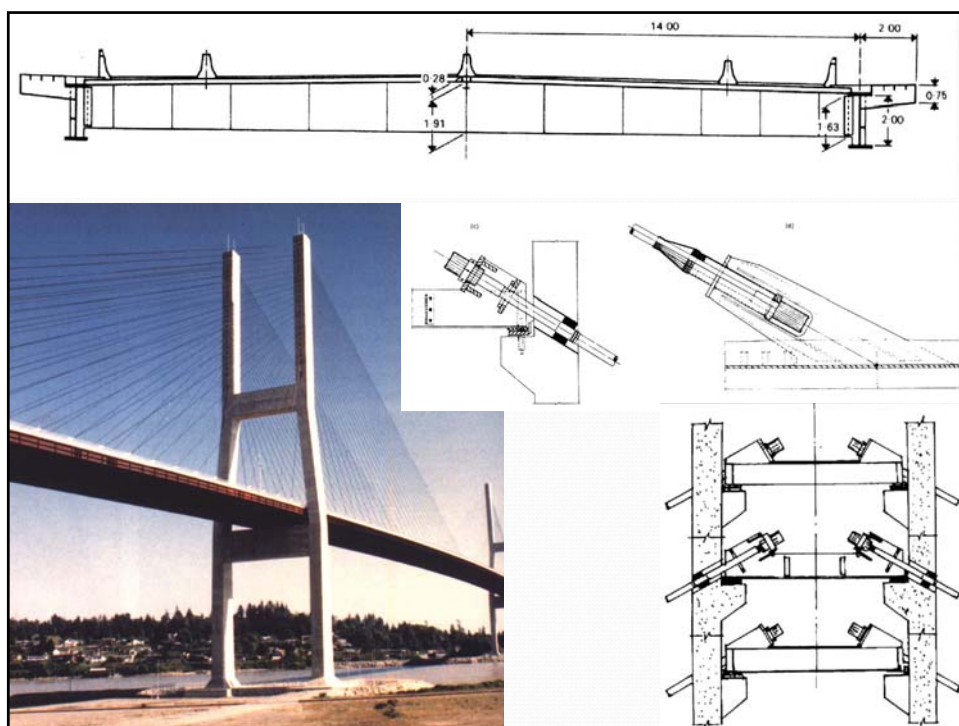
MOST PELJEŠAC





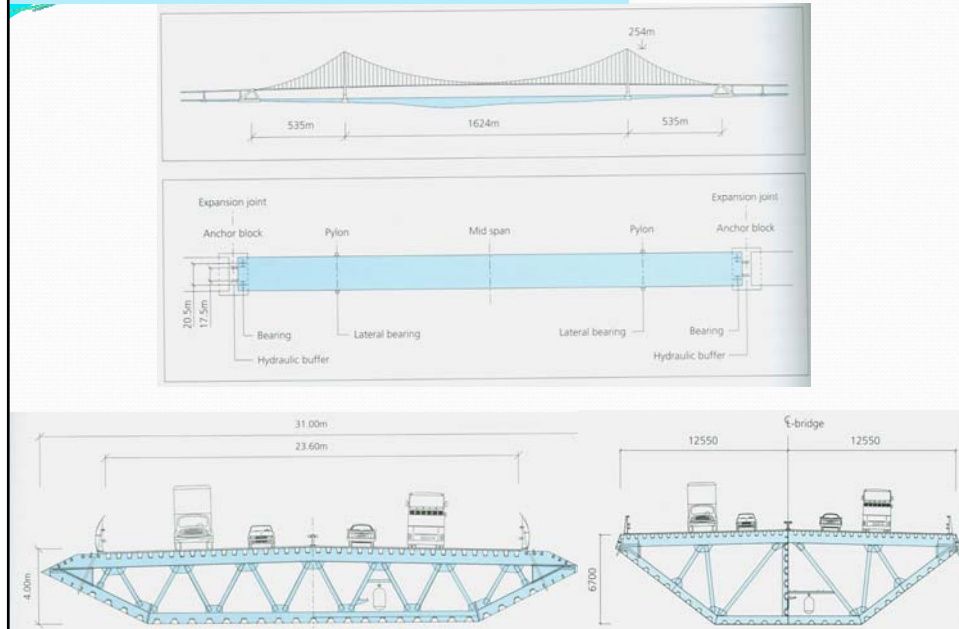






VISEĆI MOSTOVI

Veliki Belt, DANSKA, 1998.g.



TSING MA

- raspon 1377 m
- Hong Kong, Kina 1997.

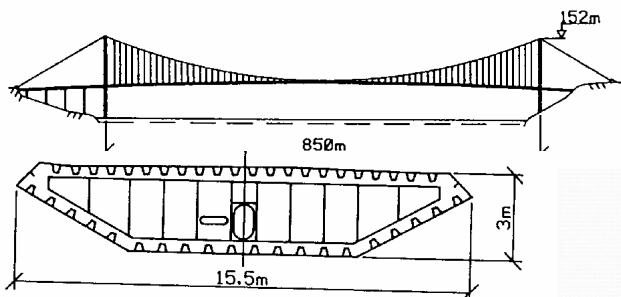
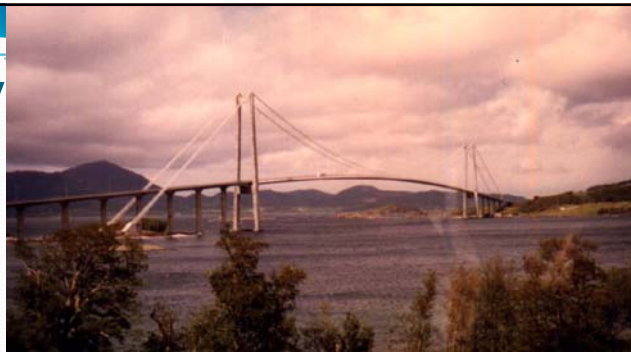


segment grede



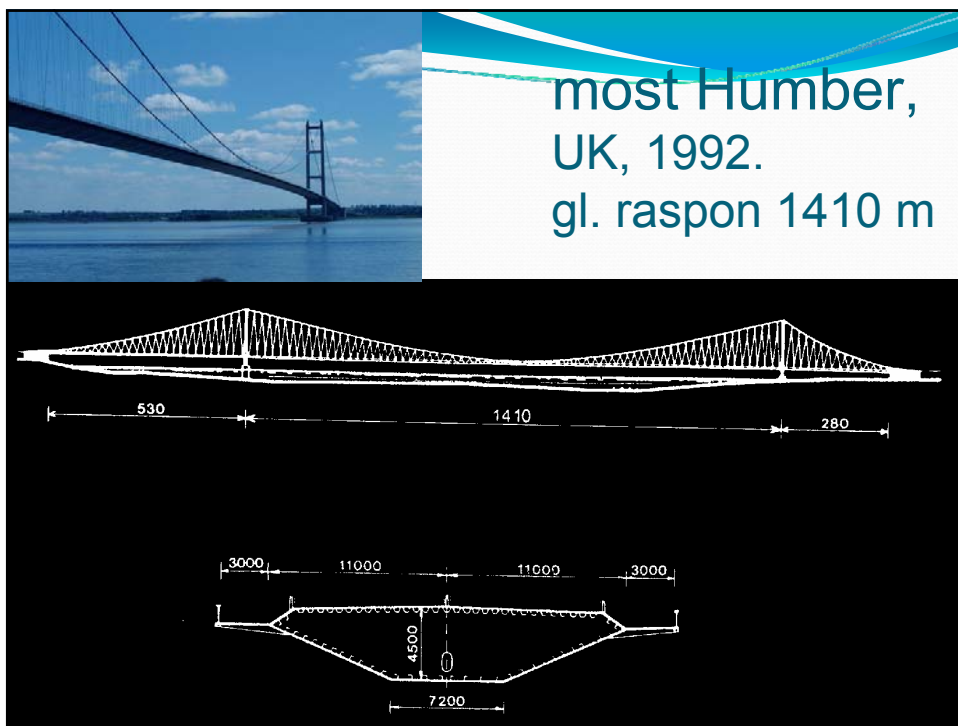


Most Askoy
 Norveška 1992
 gl. raspon: 850 m

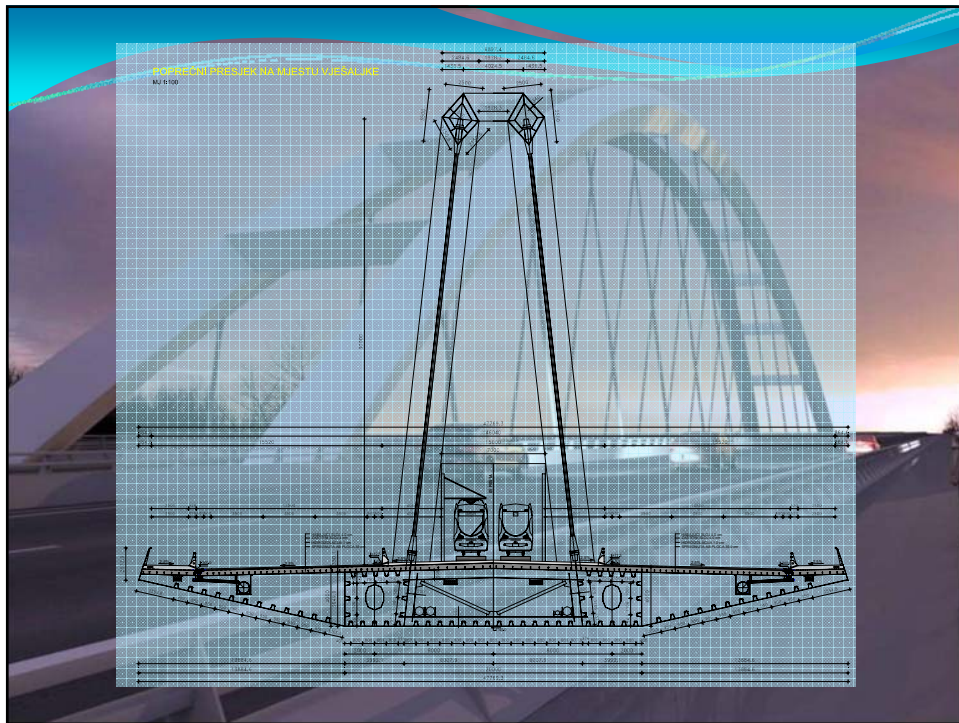
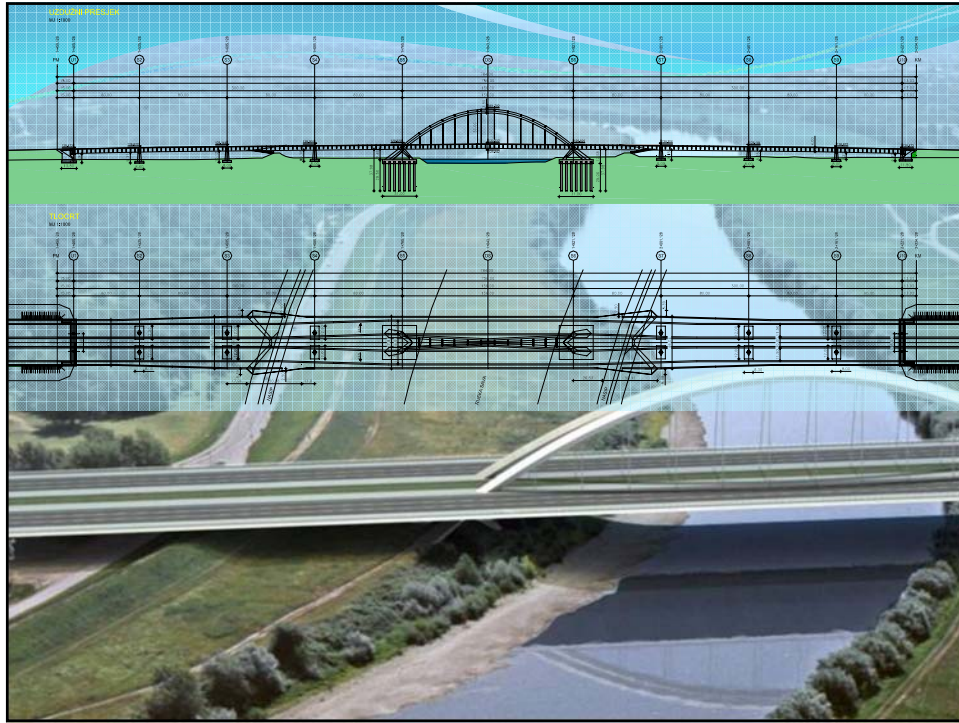


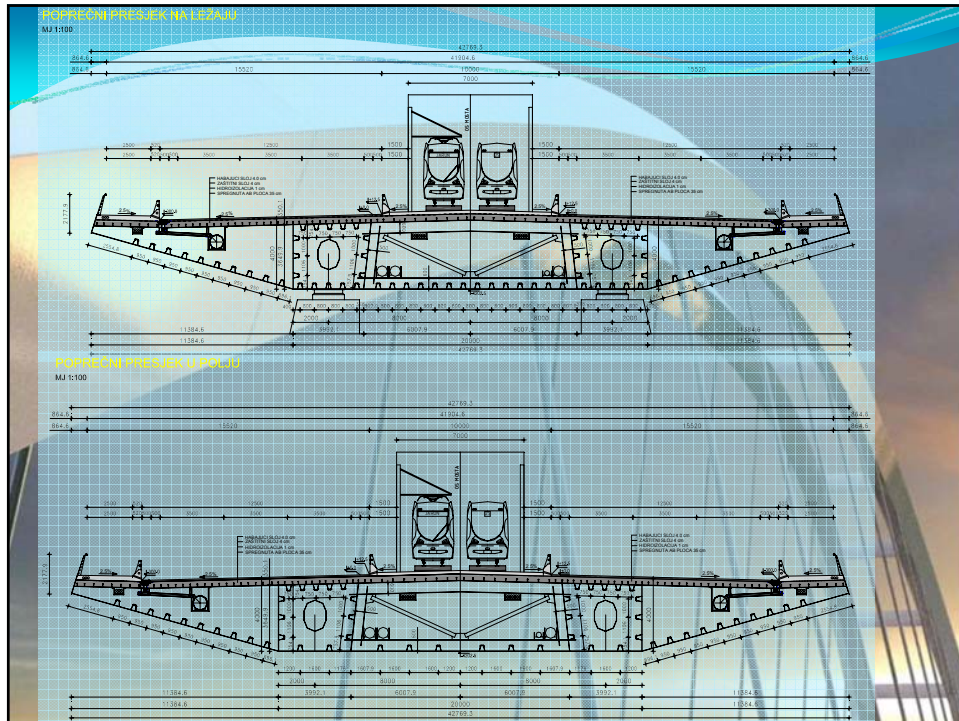
pogled

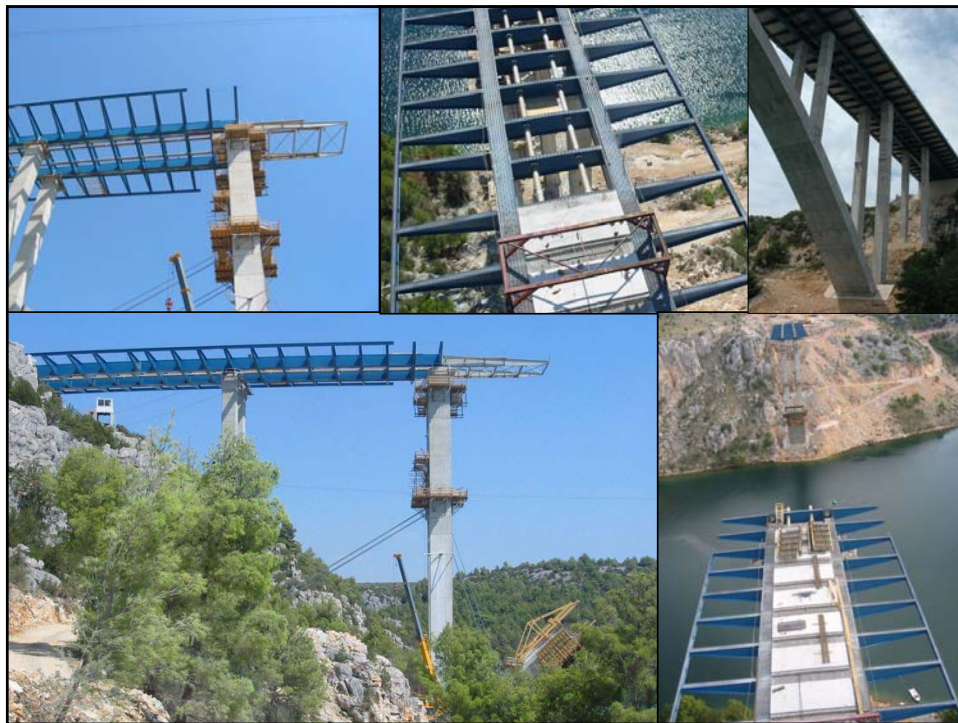
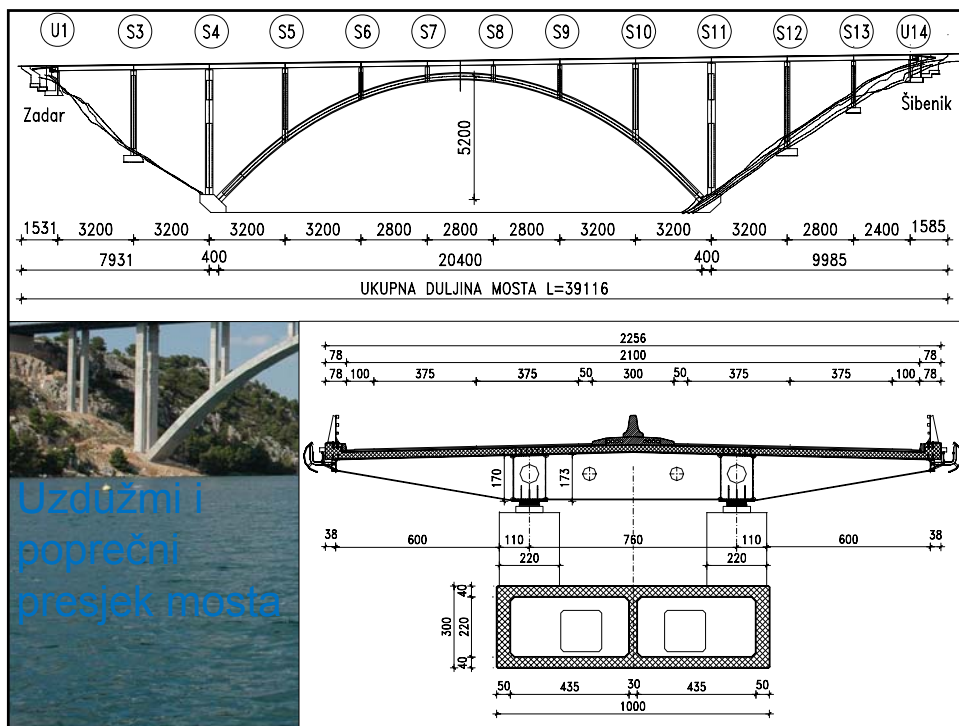
poprečni presjek



LUČNI MOSTOVI

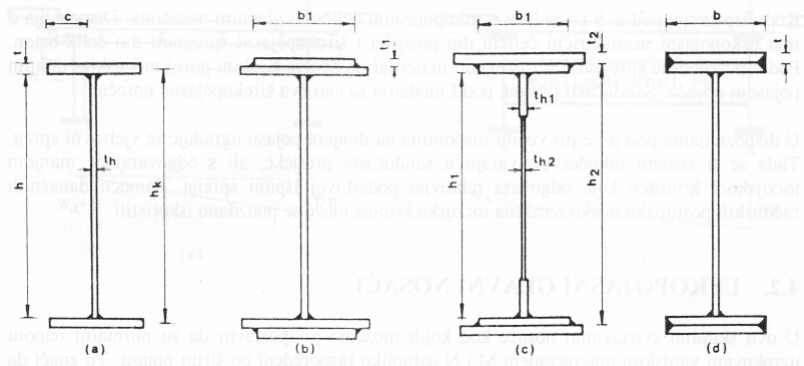




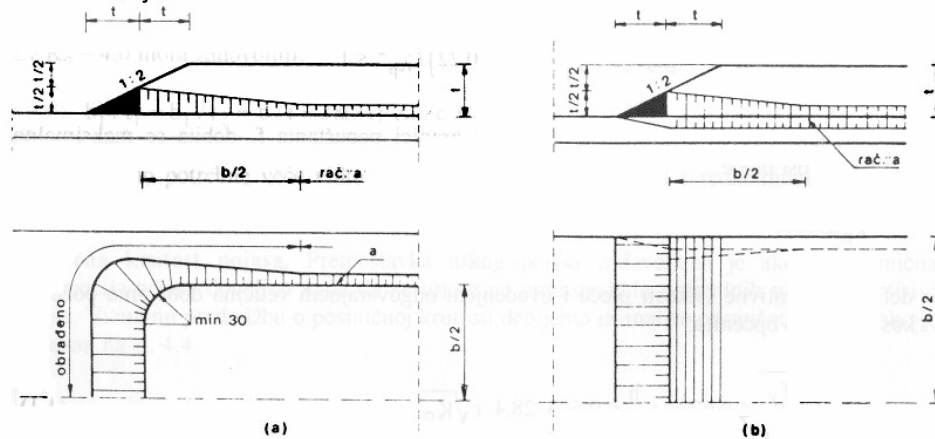


USKOPOJASNI GLAVNI NOSAČI

poprečni presjeci $\begin{cases} \rightarrow \text{lokalna stabilnost} \\ \rightarrow \text{posmična deformacija} \end{cases}$



Važni detalji za mostove:

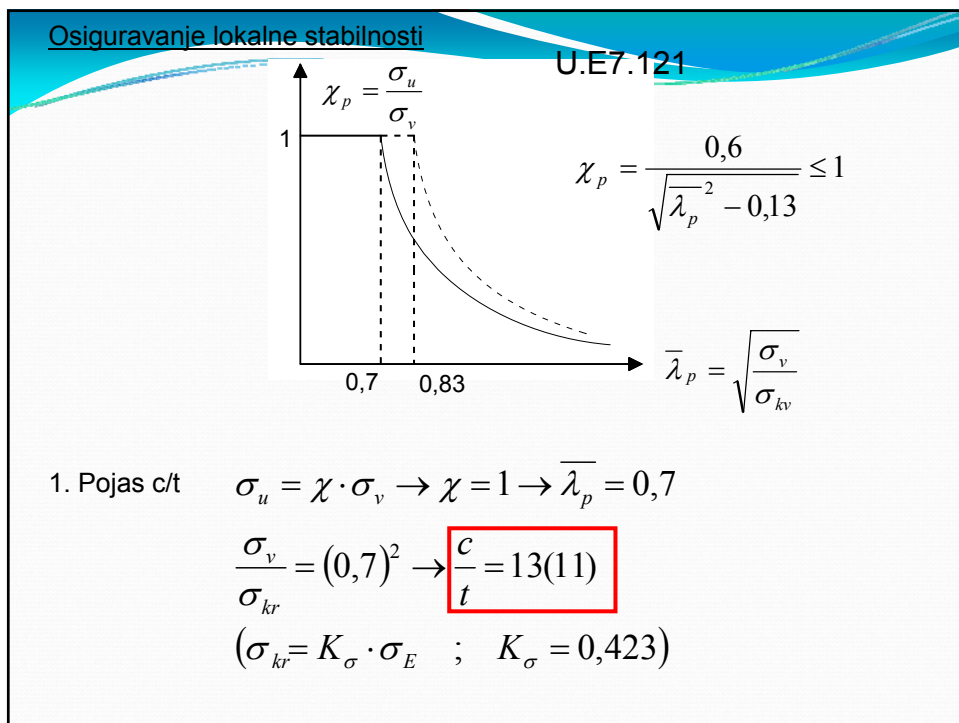
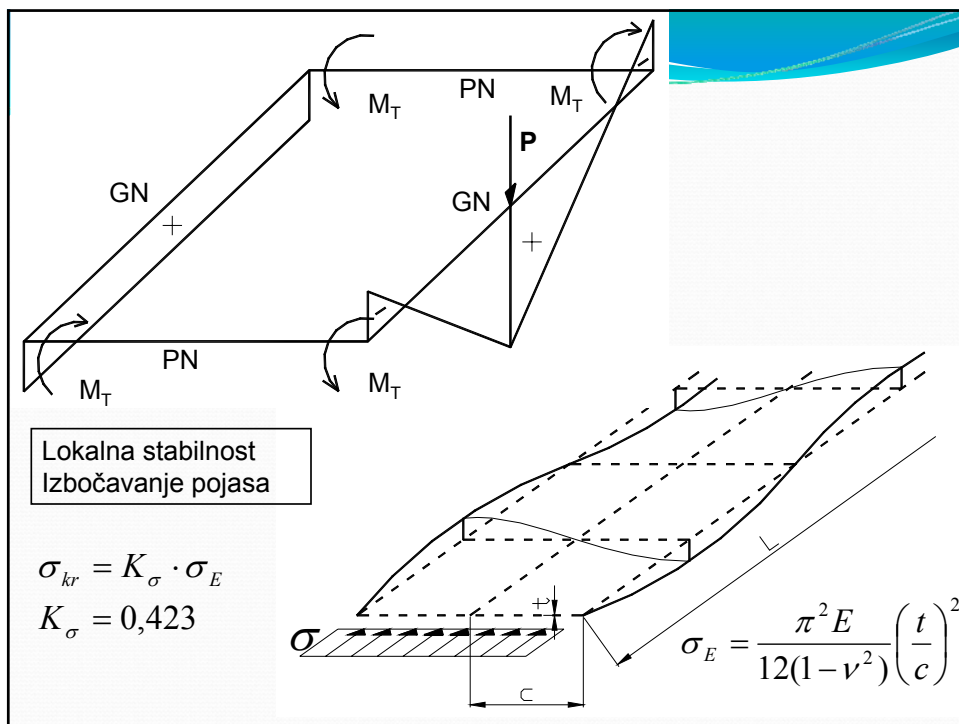


Dimenzije:
$$h_k = \frac{L}{10} - \frac{L}{12} \quad \text{do} \quad \frac{L}{14} - \frac{L}{16} \quad (\text{željez. most.})$$

$$L < 20\text{m} \quad L > 20\text{m}$$

$t_h = 12, 13, 14$ za optimalne visine

$$h_k = \frac{L}{15} - \frac{L}{30} \quad (\text{cestovni mostovi})$$



2. Pojas b_o/t $\chi = 1$ za $\overline{\lambda_p} = 0,7$ $K_\sigma = 4,0$

$$\frac{\sigma_u}{\sigma_{kv}} = (0,7)^2 \rightarrow \frac{b_o}{t} = 39(32)$$

3. Hrbat h/t $\sigma_u = 1,25 \cdot \chi \cdot \sigma_v$

$$\overbrace{1,0} \rightarrow \overline{\lambda_p} = 0,83$$

$$K_{\sigma} = 23,9 \quad do \quad 7,64$$

$$\frac{\sigma_v}{\sigma_{kv}} = (0,83)^2 \rightarrow \frac{h}{t} = 114(94)$$

$$\sigma_{kr} = \frac{\pi^2 EI}{l^2 A}$$

$$I = \frac{bt^3}{12} \quad A = bt$$

$$\sigma_{kr} = \frac{\pi^2 E}{12(1-\mu^2)} \left(\frac{t}{l} \right)^2 \cdot K = \sigma_E \cdot K_\sigma$$

korekcija $(1 - \mu^2)$ – za ploču

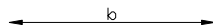
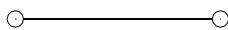
KOEFICIJENTI IZBOČAVANJA: "k"

σ  $K_\sigma = 0,423$

σ  $K_\sigma = 4,0$

$$\overline{\lambda}_p = \sqrt{\frac{\sigma_v}{\sigma_{kr}}}$$

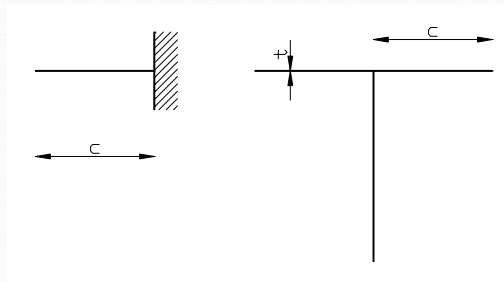
$$\sigma \left[\text{mm} \right]^{-1} K_{\sigma} = 23.9$$



$$\overline{\lambda}_p = \sqrt{\frac{\sigma_v}{\sigma_{kr}}}$$

$$\sigma_{kr} = \sigma_E \cdot K_\sigma = \frac{\pi^2 E}{12(1-\mu^2)} \cdot \left(\frac{t}{b}\right)^2 \cdot K_\sigma$$

POJAS



$$K_\sigma = 0,423$$

maksimalna vitkost pojasa c/t

granična nosivost f_u

$$f_u = \kappa_p \cdot f_y \quad \kappa_p = (\overline{\lambda}_p - 0,22) / \overline{\lambda}_p^2 \leq 1$$

κ_p ...prema osnovnoj krivulji izbočavanja

$$\kappa_p = 1 \Rightarrow \overline{\lambda}_p = \overline{\lambda}_p - 0,22$$

ili

$$\overline{\lambda}_p^2 - \overline{\lambda}_p + 0,22 = 0$$

$$\overline{\lambda}_p = 0,5 \pm \sqrt{0,5^2 - 0,22}$$

$$\overline{\lambda}_p \leq 0,673$$

DEFINICIJA

$$\overline{\lambda}_p = \sqrt{\frac{f_y}{\sigma_{kr}}}$$

$$\sigma_{kr} = \frac{\pi^2 E}{12(1-\mu^2)} \cdot \left(\frac{t}{b}\right)^2 \cdot K_\sigma = \frac{\pi^2 \cdot 2,1 \cdot 10^5}{12 \cdot (1-0,3^2)} \cdot \left(\frac{t}{b}\right)^2 \cdot K_\sigma = 189800 \left(\frac{t}{b}\right)^2 \cdot K_\sigma$$

$$\overline{\lambda_p} = 0,673 \quad \dots \quad \text{granična vitkost} \Rightarrow f_u = f_y$$

$$\overline{\lambda}_p = \sqrt{\frac{f_y}{\sigma_{kr}}} = \sqrt{\frac{f_y}{189800 \cdot \left(\frac{t}{b}\right)^2 \cdot K_\sigma}}$$

$$\overline{\lambda_p} = 0,673 = \frac{b}{t} \cdot \frac{1}{435,661} \sqrt{\frac{K_\sigma}{f_y}}$$

$$\Rightarrow \frac{b}{t} = 0,673 \cdot 435,661 \cdot \sqrt{\frac{K_{\sigma}}{f_y}} = 293,2 \sqrt{\frac{K_{\sigma}}{f_y}}$$

(a) $K_{\sigma} = 0,423$

$$Fe \quad 360 \quad \frac{b}{t} \leq 293,2 \sqrt{\frac{0,423}{235}} = 12,6$$

$$Fe \quad 510 \quad \frac{b}{t} \leq 293,2 \sqrt{\frac{0,423}{355}} = 10,1$$

(b) $K_{\sigma} = 4,0$

$$Fe \quad 360 \quad \frac{b}{t} \leq 293,2 \sqrt{\frac{4,0}{235}} = 38,2$$

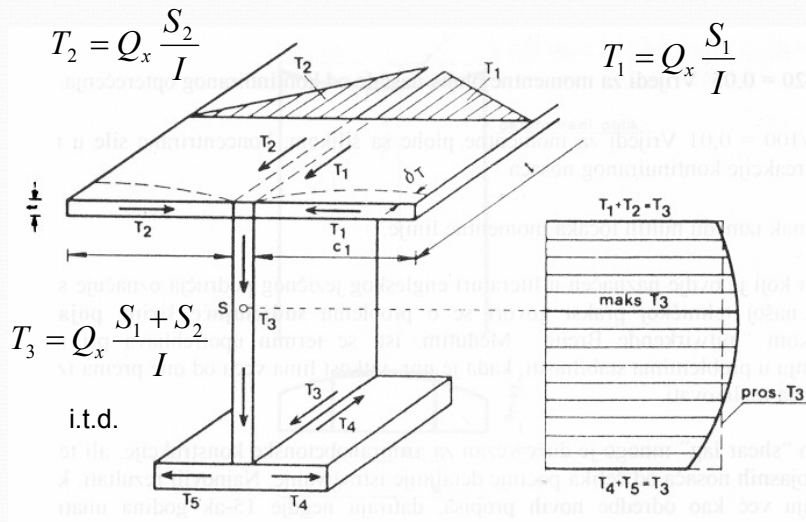
$$Fe \quad 510 \quad \frac{b}{t} \leq 293,2 \sqrt{\frac{4,0}{355}} = 31,1$$

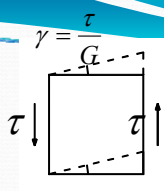
(c) $K_\sigma = 23,9$

$$Fe \quad 360 \quad \frac{b}{t} \leq 293,2 \sqrt{\frac{23,9}{235}} = 93,5$$

$$Fe \quad 510 \quad \frac{b}{t} \leq 293,2 \sqrt{\frac{23,9}{355}} = 76,1$$

POSMIČNA KRUTOST I POSMIČNI TOK



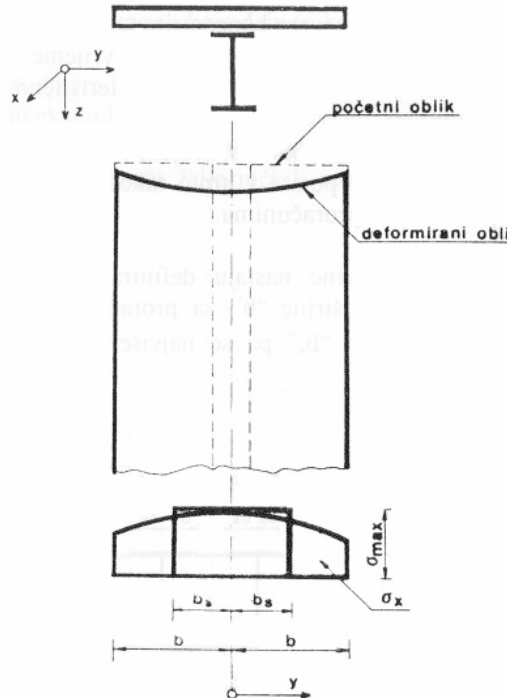


Deformacija $\gamma = \frac{T}{t \cdot G} \dots \gamma \cdot c \equiv \delta_T$

“p” $\frac{c}{l} \leq \frac{1}{20} \dots \delta_T \approx 0 \equiv \text{uski pojas}$

“P” $\frac{c}{l} \leq \frac{1}{100}$

$\frac{c}{l} > \frac{1}{20} = \text{široki pojas}$



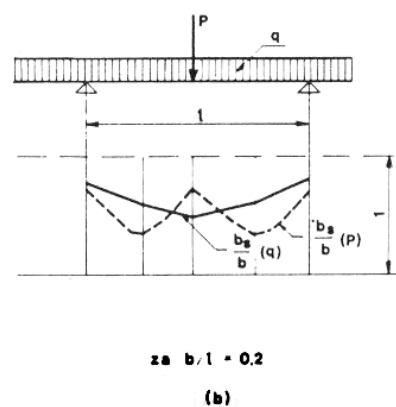
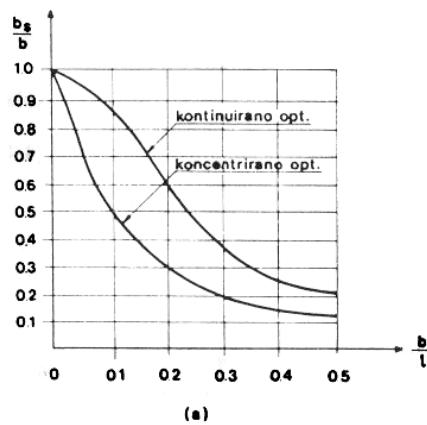
$$b_s \cdot \sigma_{\max} = \int_0^b \sigma_x dy$$

b_s = sudjelujuća ili efektivna širina od [Shear lag](#)

Shear lag ovisi o:

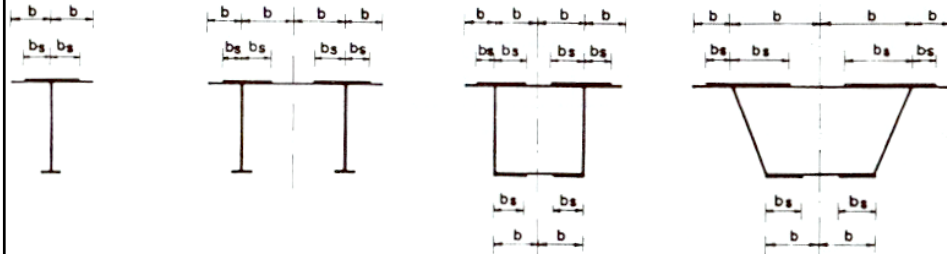
- b/L , c/L - osnovni parametar
 - p , P - vrsta opterećenja
 - stupnju ortotropije ploče
 - rasporedu opterećenja poprečno
 - vrsti materijala E , G , ν
 - odnosu A_p/A_h
- mali utjecaj {

Utjecaj vrste opterećenja

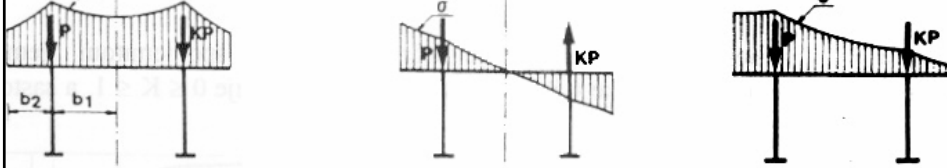


DEFINIRANI RAČUNSKI PRESJECI

simetrično opterećenje



nesimetrično opterećenje



$K=1$

$$\alpha \cdot b_{1s}; \quad b_2 = b_{2s}$$

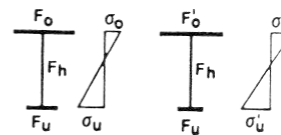
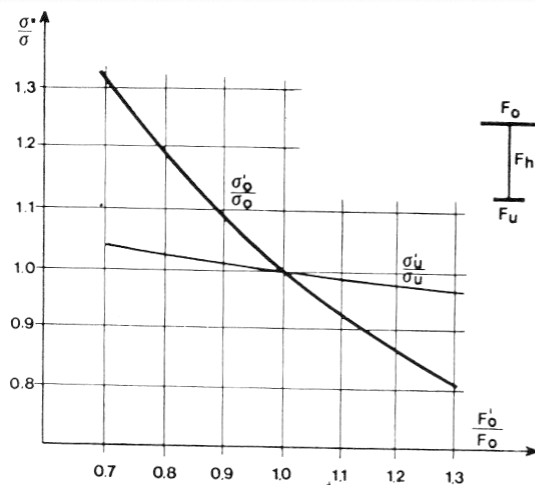
$K=1$

$0 < K < 1$

$$\alpha = 1 - 0,5 \cdot \frac{b_{1s}}{b_1} \left(1 - \frac{b_2}{b_1} \right) \cdot [1 - 0,64K - 0,36K^2]$$

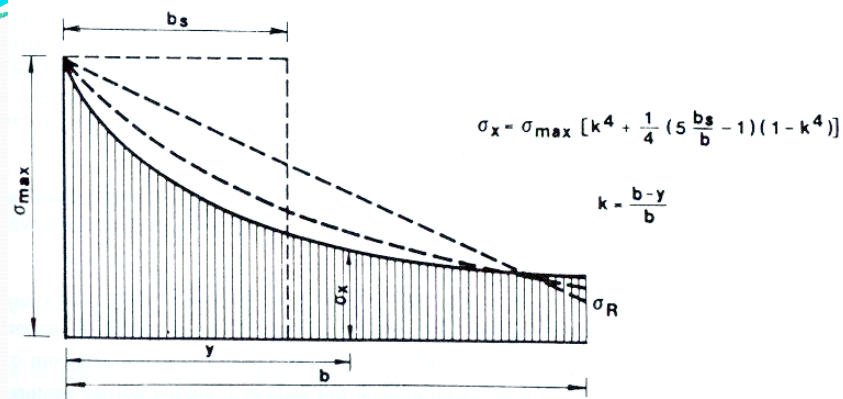
Utjecaj pogreške u sudjelujućoj površini

F_o = ispravna površina; F_o' = pogrešna površina



$F_o = 200$
 $F_u = 100$
 $F_h = 160$

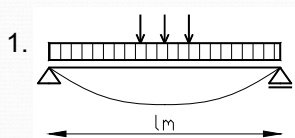
Superpozicija od spregnutog i lokalnog djelovanja



- pravac – na strani najveće sigurnosti
- parabola 2. reda – povoljnija
- parabola 4. reda – BS 5460

SUDJELUJUĆA ŠIRINA PREMA DIN 18809

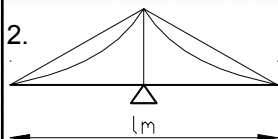
$$b_m = \lambda \cdot b$$



$$\lambda_1 = \frac{1}{1 + 6,4 \left(\frac{b}{l_m} \right)^2}$$

$$\frac{b}{l_m} = \frac{l}{20} \quad \lambda_1 = 1$$

- za polja prostih greda i kontinuiranih nosača



$$\lambda_2 = \frac{1}{1 + 6 \left(\frac{b}{l_m} - 1/2500 \frac{b}{l_m} \right) + 1,6 \left(\frac{b}{l_m} \right)^2} \quad \text{za } \frac{b}{l_m} \geq \frac{1}{20}$$

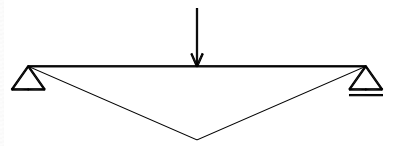
za $\frac{1}{20} > \frac{b}{l_m} \geq \frac{l}{50}$ $\lambda_2 = \text{linearna interpolacija između } 0,767 - 1,0$

za $\frac{b}{l_m} < \frac{l}{50}$ $\lambda_2 = 1,0$

- za oslonce kontinuiranih nosača i konzole

SUDJELUJUĆA ŠIRINA PREMA DIN 18809

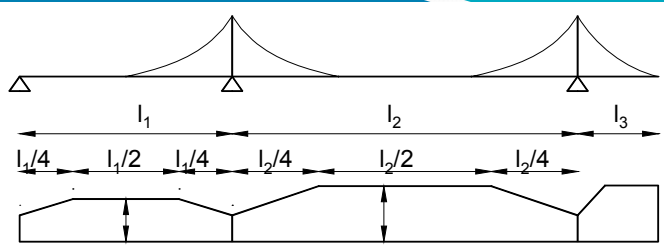
3.



$$\lambda_3 = \frac{1}{1 + 4\left(\frac{b}{l_m}\right) + 3,2\left(\frac{b}{l_m}\right)^2} \quad \text{za} \quad \frac{b}{l_m} \geq \frac{1}{20}$$

$$\text{za} \quad \frac{1}{20} > \frac{b}{l_m} \geq \frac{1}{50} \quad \lambda_2 = \text{linearna interpolacija između } 0,828 - 1,0$$

- za montažna stanja i popuštanje oslonaca

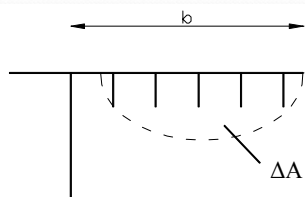


- kod ortotropnih ploča sa uzdužnim rebrima u navedene izraze ulazi se s povećanom širinom b' :

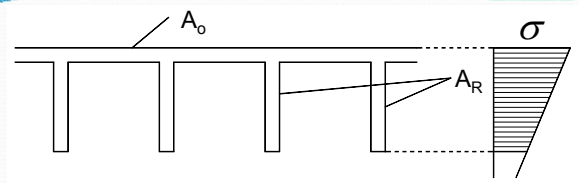
$$b' = \alpha \cdot b$$

$$\alpha = \sqrt{1 + \frac{\Delta A}{bt}}$$

ΔA = ploha uzdužnih rebara na širini b



Utjecaj ortotropije



$$\sigma \dots EI \dots \left\{ \begin{array}{l} A_o = bt \\ A_R = rebro \end{array} \right\} A_o + \sum A_R$$

$$\tau \dots GA \dots \left\{ \begin{array}{l} A_o = bt \\ A_R \approx 0 \end{array} \right\} A \approx A_o$$

$$\delta_T^{ORTOTROPNO} > \delta_T^{IZOTROPNO}$$

$$b_S^{ORTOTROPNO} < b_S^{IZOTROPNO}$$

Praktički:

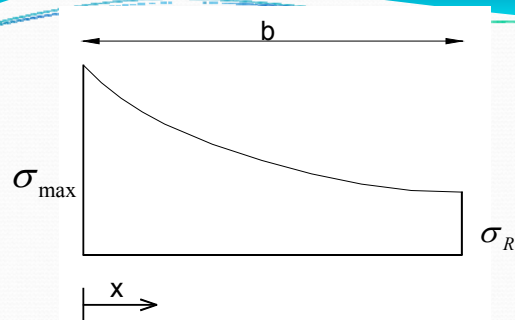
$$\frac{\sum A_R}{b \cdot t} = 0 \rightarrow \text{izotropna pločl}$$

Propisi:

$$0 < \frac{\sum A_R}{bt} < 1 \rightarrow \text{ortotropna pločl}$$

Stvarno područje primjene:

$$0,25 < \frac{\sum A_R}{bt} < 2,3$$

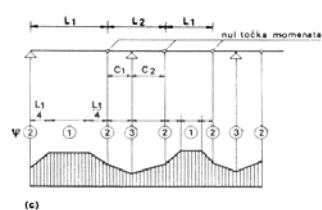
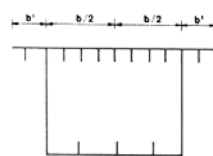
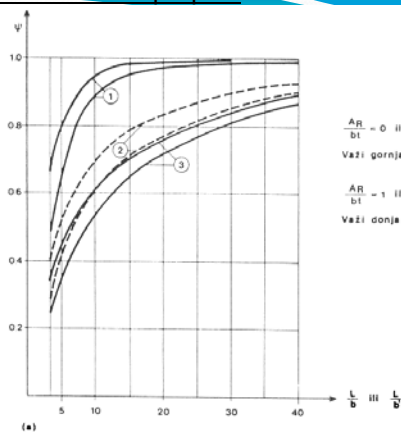


$$\sigma_R = 1,25 \left(\frac{b_s}{b} - 0,2 \right) \cdot \sigma_{\max} \geq 0$$

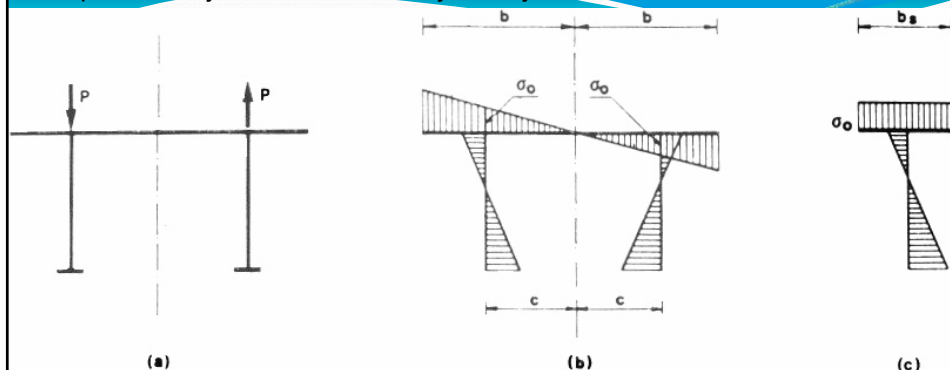
$$\sigma_x = (\sigma_{\max} - \sigma_R) \cdot \left(1 - \frac{x}{b} \right)^4 + \sigma_R$$

Primjer iz prijedloga za američke propise

$$b_s = \psi \cdot b$$



Aproksimacija za antimetrično djelovanje



Uvjet: $N \cdot a = \sigma_o \cdot b_s \cdot 2c$

$\downarrow$ $\downarrow$
 stvarno zamjenjujuće

$$\underbrace{\sigma_o \cdot \frac{b}{c} \cdot \frac{b}{2}}_N \cdot \underbrace{\left(\frac{2}{3} \cdot 2 \cdot b \right)}_a = \sigma_o \cdot b_s \cdot 2c$$

$$b_s = \frac{b}{3} \cdot \left(\frac{b}{c} \right)^2$$

Vrijedi za $\frac{b}{c} \leq 1/5$ (ne prevelike konzole)

Točnije antimetrično – torzijsko opterećenje

Model: Simetrično + torzijsko = Nesimetrično



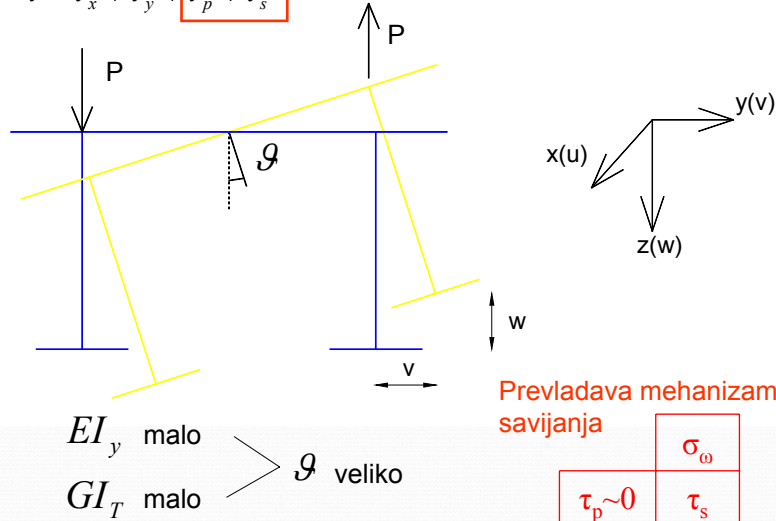
TORZIJA

$$\sigma = \sigma_N + \sigma_x + \sigma_y + \sigma_\omega$$

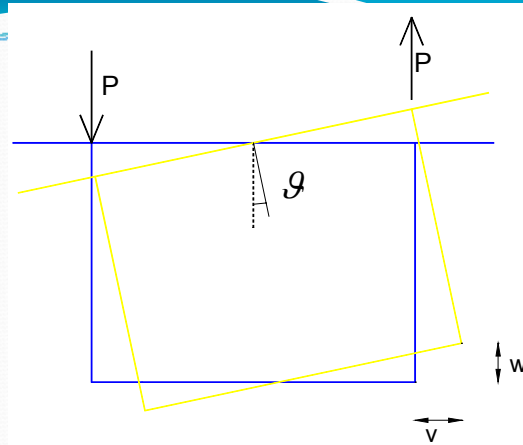
$$\tau = \tau_x + \tau_y + \tau_p + \tau_s$$

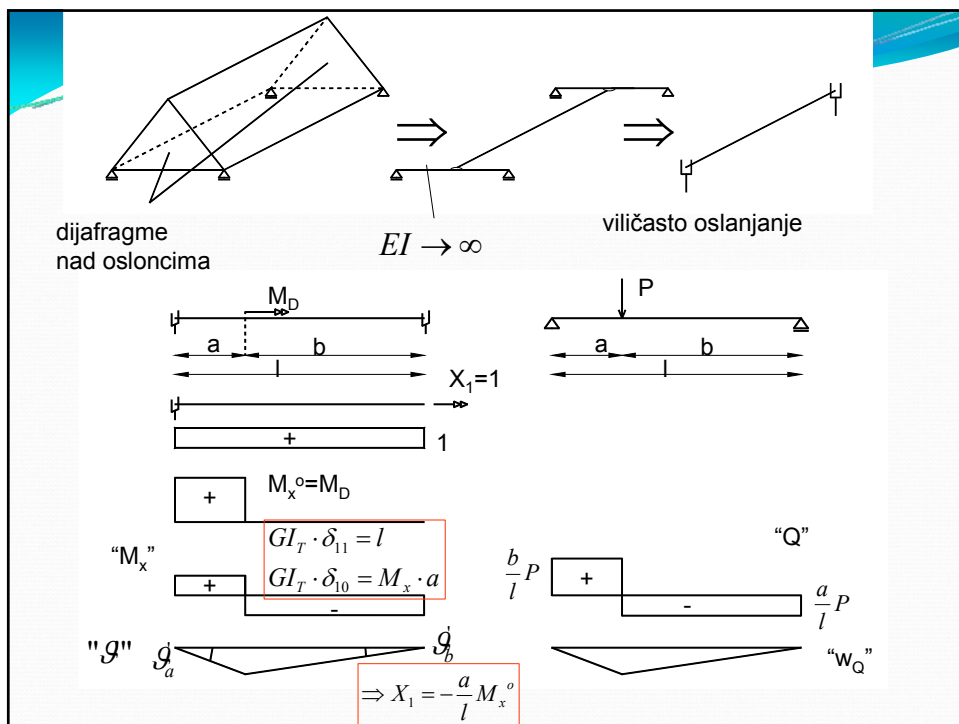
ograničena torzija ili
krivljenje poprečnog presjeka
 M_ω

1.



2.





DIFERENCIJALNA JEDNADŽBA ST. VENANTOVE TORZIJE

Diagram of a rectangular cross-section with width b and height t , showing shear stress distribution τ and radius r .

Key equations and steps:

- $v' = \frac{M_D}{GI_D} = \text{const.}$
- $\tau_r = \frac{2\tau_{\max}}{t} \cdot r$
- $dM_D = 4\tau_r dr \cdot r \cdot b = 8 \frac{\tau_{\max}}{t} t^2 b dr$
- VANJSKI PRSTEN $r = t/2$
- $dM_D = 2bt\tau_{\max} dr$
- za $b \gg t$ $dM_D v' = \frac{1}{G} 2b\tau_{\max}^2 dr$
- $\Rightarrow \tau_{\max} = Gt v'$
- $M_D = \frac{8\tau_{\max}}{t} b \int_0^{t/2} r^2 dt = \frac{1}{3} \tau_{\max} b t^2$
- $M_D = v' G \frac{1}{3} b t^3 = GI_D v'$
- $\Rightarrow I_D = b t^3 / 3$
- max $\tau = \frac{3M_D}{b t^2} = \frac{M_D}{I_D} t$
- $\gamma = \frac{\tau}{G}$ $T = \tau \cdot t$

$$\sum M_D = 0$$

$$M_D = \oint r_i T ds = T \oint r_i ds$$

$$\oint r_i ds = 2F_m$$

$$\Rightarrow T = \frac{M_D}{2F_m} \quad \max \quad \tau_p = \frac{T}{t_{\min}} \quad (1.BREDT)$$

JEDNADŽBA RADA

$$\frac{1}{2} M_D \cdot dv = \frac{1}{2} \oint \tau_p t \gamma dx ds$$

$$\gamma = \tau_p / G$$

$$M_D \cdot dv = dx \oint \frac{\tau_p^2}{G} t ds = dx \frac{T^2}{G} \oint \frac{ds}{t}$$

$$v' = \frac{M_D}{4GF_m^2} \oint \frac{ds}{t}$$

$$v' = \frac{M_D}{GI_D} \Rightarrow I_D = \frac{4F_m^2}{\oint \frac{ds}{t}} \quad (2.BREDT)$$

$$\Rightarrow \tau = \frac{T}{t} = \frac{M_T}{2A_m} \cdot \frac{1}{t} = \frac{GI_T v'}{2A_m t} = G \frac{\psi}{t} v'$$

$$\tau = G \frac{\psi}{t} \cdot \frac{M_T}{GI_T} = \frac{M_T}{I_T} \cdot \frac{\psi}{t}$$

analogija $\sigma = \frac{M_S}{I} \cdot y$

$$M_T = GI_T v' = \int \tau \left(\frac{\psi}{t} \right) dA = \int G \left(\frac{\psi}{t} \right) \cdot v' \left(\frac{\psi}{t} \right) dA = G v' \int \left(\frac{\psi}{t} \right)^2 dA$$

$$\Rightarrow I_T = \int \left(\frac{\psi}{t} \right)^2 dA \quad \dots \quad I_S = \int y^2 dA$$

VIŠEĆELIJSKI SUSTAVI: $M_T = \sum 2A_{mj} T_j \quad j = \text{broj ćelije}$

$$v' = v'_1 = v'_2 = v'_3 = \frac{T \cdot 2A_{mj}}{GI_T} = \frac{1}{2GA_{mj}} \oint T \frac{ds}{t}$$

ili

$$v' = \frac{T}{G\psi}$$

za j -ćeliju:

$$Gv' = \frac{1}{2A_{mj}} \left(-T_{j-1} \int_a^b \frac{ds}{t} + T_j \oint \frac{ds}{t} - T_{j+1} \int_c^d \frac{ds}{t} \right)$$

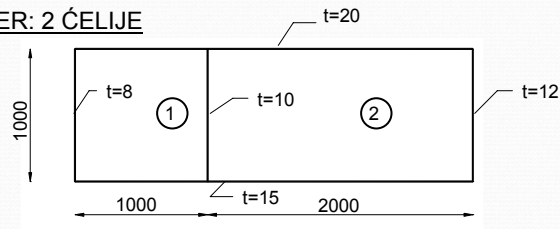
⇒ SUSTAV JEDNADŽBI ZA N-ČELIJA

1 RAVNOTEŽA
N DEFORMACIJSKIH UVJETA

N+1 ZA "N" NEPOZNATIH POSMIČNIH TOKOVA I v'

$$\text{IZ } v' = \frac{M_T}{GI_T} \Rightarrow I_T$$

PRIMJER: 2 ČELIJE



$$G = 8100 \text{ kN/cm}^2$$

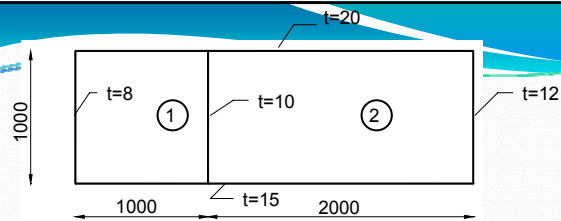
$$M_T = 1000 \text{ kNm}$$

$$A_{m1} = 100 \cdot 100 = 10000 \text{ cm}^2$$

$$A_{m2} = 100 \cdot 200 = 20000 \text{ cm}^2$$

$$2GA_{m1} = 2 \cdot 8100 \cdot 10000 = 1,62 \cdot 10^8 \text{ kN}$$

$$2GA_{m2} = 2 \cdot 8100 \cdot 20000 = 3,24 \cdot 10^8 \text{ kN}$$



$$\oint_1 \frac{ds}{t} = \frac{100}{0,8} + \frac{100}{2,0} + \frac{100}{1,0} + \frac{100}{1,5} = 341,67$$

$$\oint_2 \frac{ds}{t} = \frac{100}{1,0} + \frac{200}{2,0} + \frac{200}{1,5} + \frac{100}{1,2} = 416,67$$

$$\int_{1,2} \frac{ds}{t} = \frac{100}{1,0} = 100,0$$

$$341,67T_1 - 100T_2 - 1,62 \cdot 10^8 v' = 0$$

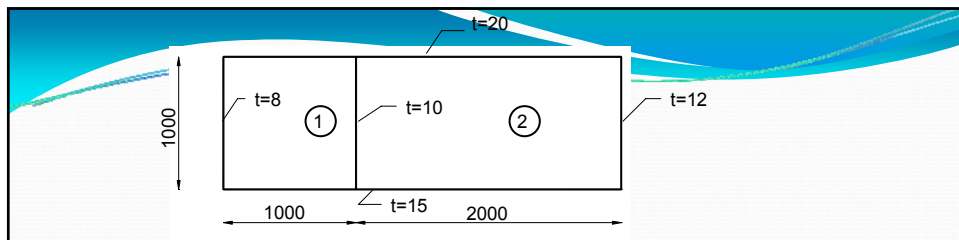
$$-100T_1 + 416,67T_2 - 3,24 \cdot 10^8 v' = 0$$

$$20000T_1 + 40000T_2 = 100000$$

RJEŠENJE: $T_1 = 1,41221 \text{ kN/cm}$ $T_2 = 1,79389 \text{ kN/cm}$

$$v' = 0,18711 \cdot 10^{-5} \text{ 1/cm}$$

$$\Rightarrow I_T = \frac{M_T}{Gv'} = \frac{100000}{8100 \cdot 0,18711 \cdot 10^{-5}} = 6598036 \text{ cm}^4 = 659,8036 \text{ m}^2 \text{ cm}^2$$

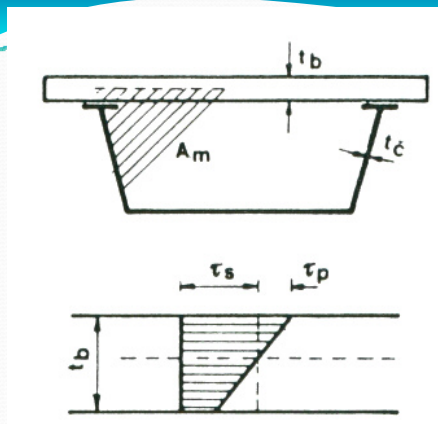


KONTROLA – BEZ UNUTRAŠNJEG HRPTA

$$2A_m = 2 \cdot 100 \cdot 300 = 60000 \text{ cm}^2$$

$$T = \frac{M_T}{2A_m} = \frac{100000}{60000} = 1,67 \text{ kN / cm}$$

$$I_T = \frac{4A_m^2}{\oint \frac{ds}{t}} = \frac{60000^2}{\left(\frac{300}{1,5} + \frac{300}{2,0} + \frac{100}{0,8} + \frac{100}{1,2} \right)} = \frac{60000^2}{558,33} = 6447761 \text{ cm}^4 = 644,776 \text{ m}^2 \text{ cm}^2$$



(s – sanduk, p – ploča)

$$M_T = M_T^s + M_T^p$$

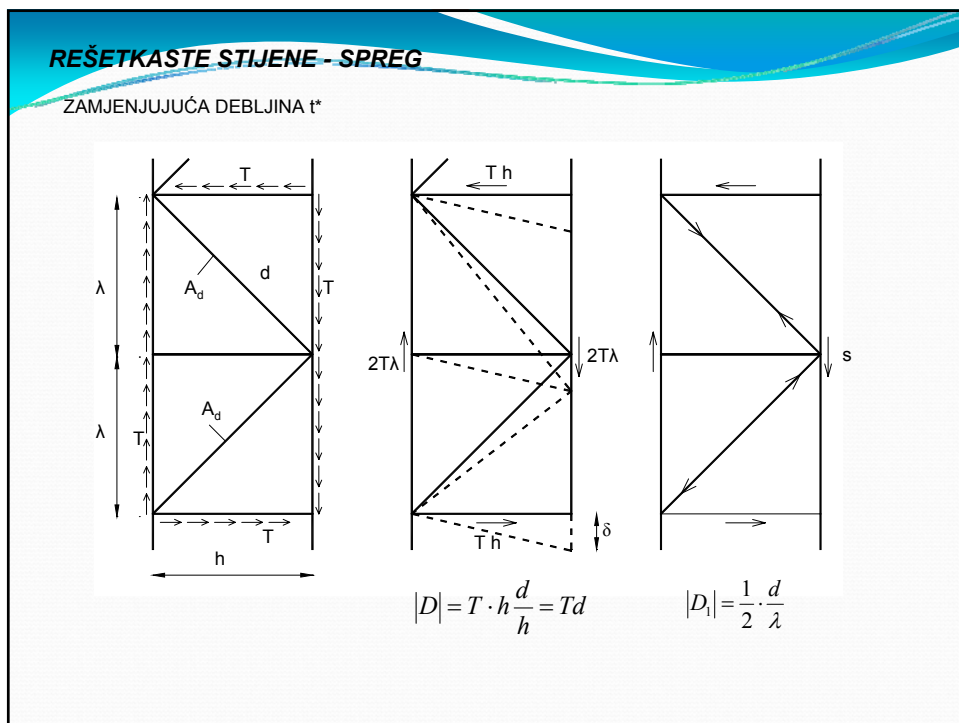
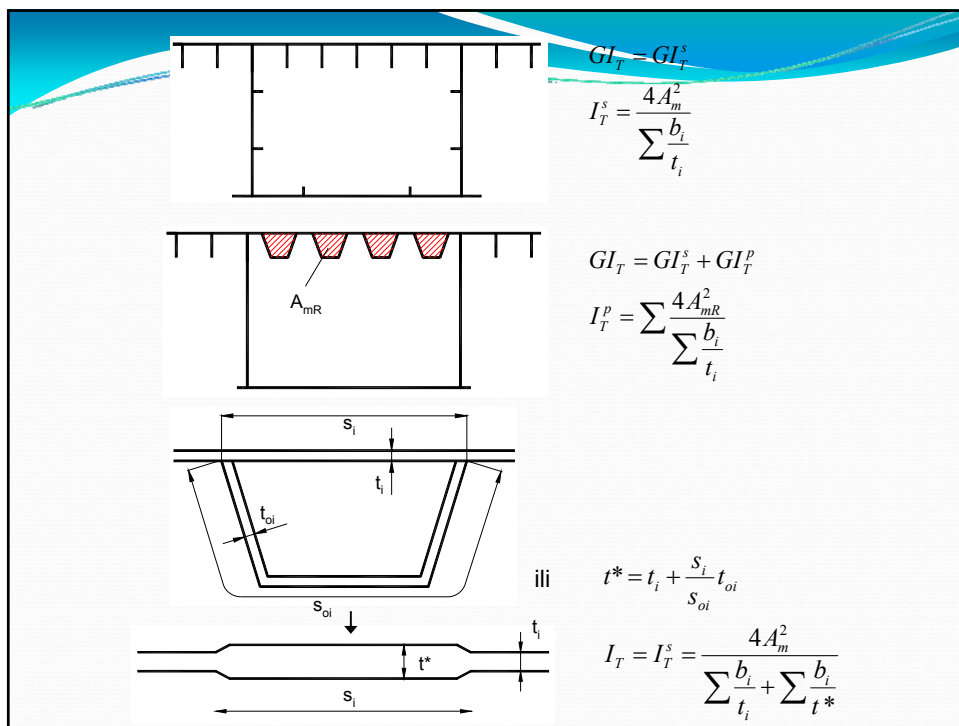
$$I_T = I_T^s + \frac{1}{n_G} I_T^p$$

$$M_T^s = \frac{I_T^s}{I_T} \cdot M_T$$

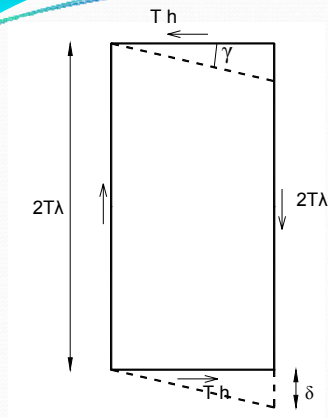
$$M_T^p = \frac{1}{n_G} \frac{I_T^p}{I_T} \cdot M_T \quad n_G = \frac{G_c}{G_b}$$

$$\tau_s = \frac{M_T^s}{2A_m} \cdot \frac{1}{t_b}$$

$$\tau_p = \pm \frac{M_T^p}{I_T^p} \cdot t_b$$



JEDNADŽBA RADA:



$$1 \cdot \delta = \sum \frac{DD_1}{EA_d} \cdot d = 2Td \cdot \frac{1}{2} \frac{d}{\lambda} \cdot \frac{d}{EA_d} =$$

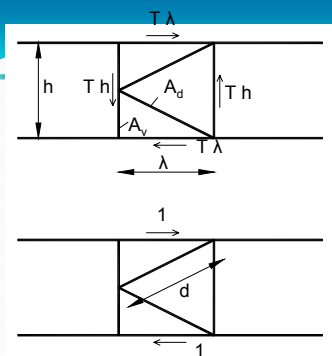
$$= \frac{Td^3}{EA_d \lambda} \quad - \text{rešetka} \quad (\text{spreg})$$

Lim debljine "t*" mora biti jednako krut na posmik kao spreg

$$\gamma = \frac{\tau}{G} = \frac{T}{Gt^*} \quad \delta = \gamma \cdot h = \frac{T \cdot h}{G \cdot t^*}$$

$$\Rightarrow \delta = \frac{T \cdot h}{G \cdot t^*} = \frac{T \cdot d^3}{EA_d \cdot \lambda}$$

$$t^* = \frac{EA_d \cdot \lambda \cdot h}{Gd^3}$$



$$V = \frac{Th}{2}$$

$$D = \frac{Th}{2} \frac{d}{h/2} = Td$$

$$D = \frac{d}{\lambda}$$

$$V = \frac{d}{\lambda} \cdot \frac{h/2}{d} = \frac{h}{2\lambda}$$

$$1 \cdot \delta = 2R_d + 2R_v = 2 \cdot \frac{Td \cdot d}{EA_d} \cdot \frac{d}{\lambda} + 2 \cdot \frac{Th}{2} \frac{h}{2EA_v} \frac{h}{2\lambda} =$$

$$= \frac{2Td^3}{EA_d \lambda} + \frac{Th^3}{4EA_v \lambda}$$

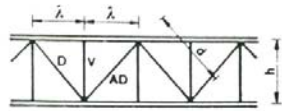
$$\gamma = \frac{T}{G} = \frac{T}{Gt^*} \quad \delta = \gamma h = \frac{Th}{Gt^*}$$

$$\frac{Th}{Gt^*} = \frac{2Td^3}{EA_d \lambda} + \frac{Th^3}{4EA_v \lambda}$$

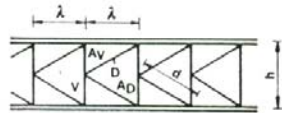
$$\Rightarrow t^* = \frac{E}{G} \cdot \frac{\lambda h}{\frac{2d^3}{A_d} + \frac{h^3}{4A_v}}$$

POJASI NISU KRUTI

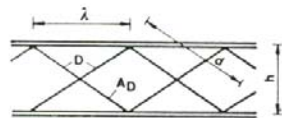
$$A_o = A_o \gg A_d$$



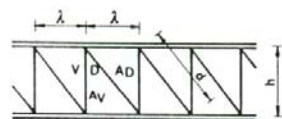
$$(a) \quad t^* = \frac{E}{G} \frac{\lambda h}{\frac{d^3}{A_d} + \frac{\lambda^3}{3} \left(\frac{1}{A_o} + \frac{1}{A_v} \right)}$$



$$(b) \quad t^* = \frac{E}{G} \frac{\lambda h}{\frac{2d^3}{A_d} + \frac{h^3}{4A_v} + \frac{\lambda^3}{12} \left(\frac{1}{A_o} + \frac{1}{A_v} \right)}$$



$$(c) \quad t^* = \frac{E}{G} \frac{\lambda h}{\frac{d^3}{2A_d} + \frac{\lambda^3}{12} \left(\frac{1}{A_o} + \frac{1}{A_v} \right)}$$



$$(d) \quad t^* = \frac{E}{G} \frac{\lambda h}{\frac{d^3}{A_d} + \frac{h^3}{A_v} + \frac{\lambda^3}{12} \left(\frac{1}{A_o} + \frac{1}{A_v} \right)}$$

TORZIJA

REKAPITULACIJA: ST. VENANT-ova (primarna torzija)

PRETPOSTAVKE:

- a) elastičan materijal
- b) mali pomaci
- c) oblik presjeka se ne mijenja
- d) torzijski momenti M_D djeluju samo na oba kraja štapa
- e) deformacije poprečnog presjeka nisu spriječene

=> NASTAJU SAMO NAPONI τ_p (St. Venant) ILI PRIMARNA TORZIJA

DIF. JEDNADŽBA ST. VENANTOVE TORZIJE:

$$\nu' = \frac{d\nu}{dx} = \frac{M_T}{GI_T} = \text{const.}$$

1. BREDT-OV IZRAZ JEDNADŽBA RAVNOTEŽE

$$M_T = 2A_m T \quad \left(M_T = T \cdot \oint r_t ds \right)$$

$$\tau = \frac{T}{t}$$

2. BREDT-OV IZRAZ

$$M_T = GI_T \frac{dv}{dx} = GI_T v'$$

$$\Rightarrow I_T = \frac{4A_m^2}{\oint \frac{ds}{t}} \quad - \text{zatvoreni presjek}$$

3. TORZIJSKA FUNKCIJA

$$\psi = \frac{T}{G \frac{dv}{dx}} = \frac{T}{G v'}$$

$$T = \frac{M_T}{2A_m} \quad v' = \frac{M_T}{GI_T} \Rightarrow \psi = \frac{M_T}{2A_m} \cdot \frac{1}{G \frac{M_T}{GI_T}} = \frac{I_T}{2A_m} = \frac{2A_m}{\int \frac{ds}{t}}$$

$$\frac{\psi}{t} = \frac{I_T}{2A_m t} = \frac{2A_m}{\int \frac{ds}{t}} \cdot \frac{1}{t}$$

TORZIJA

SAVIJANJE

$$1. \quad \tau = \frac{T}{t} = \frac{GI_T}{2A_m} \frac{v'}{t} =$$

$$= G \frac{\psi}{t} v' = \frac{M_T}{I_T} \left(\frac{\psi}{t} \right)$$

$$\sigma = \frac{M_S}{I} y$$

$$2. \quad M_T = \int \tau \left(\frac{\psi}{t} \right) dA =$$

$$M_S = \int \sigma y dA$$

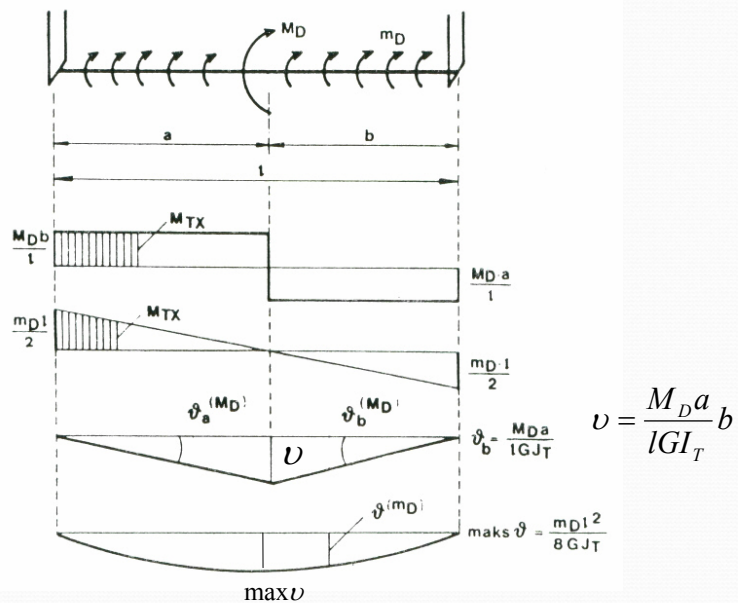
$$= \int G \left(\frac{\psi}{t} \right) v' \left(\frac{\psi}{t} \right) dA =$$

$$= GI_T v'$$

$$I_S = \int y^2 dA$$

$$I_T = \int \left(\frac{\psi}{t} \right)^2 dA$$

RASPORED TORZIJSKOG UTJECAJA



Zakretanje od torzije

$$\frac{dv}{dx} = v' = \frac{M_{Tx}}{GJ_T} \rightarrow v = \frac{M_{Tx}}{GJ_T} \int_0^a dx = \frac{M_D}{GJ_T} \frac{ba}{l}$$

$$v = \text{moment savijanja za } \frac{M_D}{GJ_T} \text{ ili } \frac{m_D}{GJ_T}$$

ili

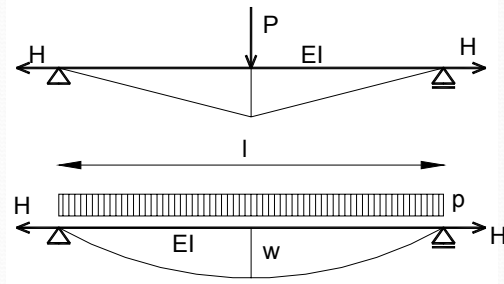
$$GJ_T v = M_v = \int_l M_{Tx} dx = \text{posmični bimoment}$$

$$\text{analogno lančanicima} \quad GJ_T \equiv H \quad v \equiv w$$

$$EI = 0$$

$$GJ_T v = M_v \equiv Hw = M_0$$

LANČANICA



$$Hw = \frac{Pl}{4}$$

$$EI = 0$$

$$Hw = \frac{pl^2}{8}$$

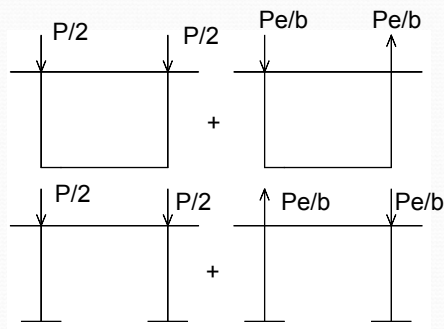
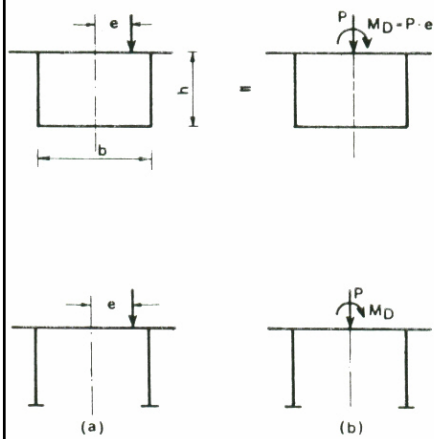
$$GI_T v = M_v$$

$$v = \frac{M_v}{GI_T}$$

posmični bimoment

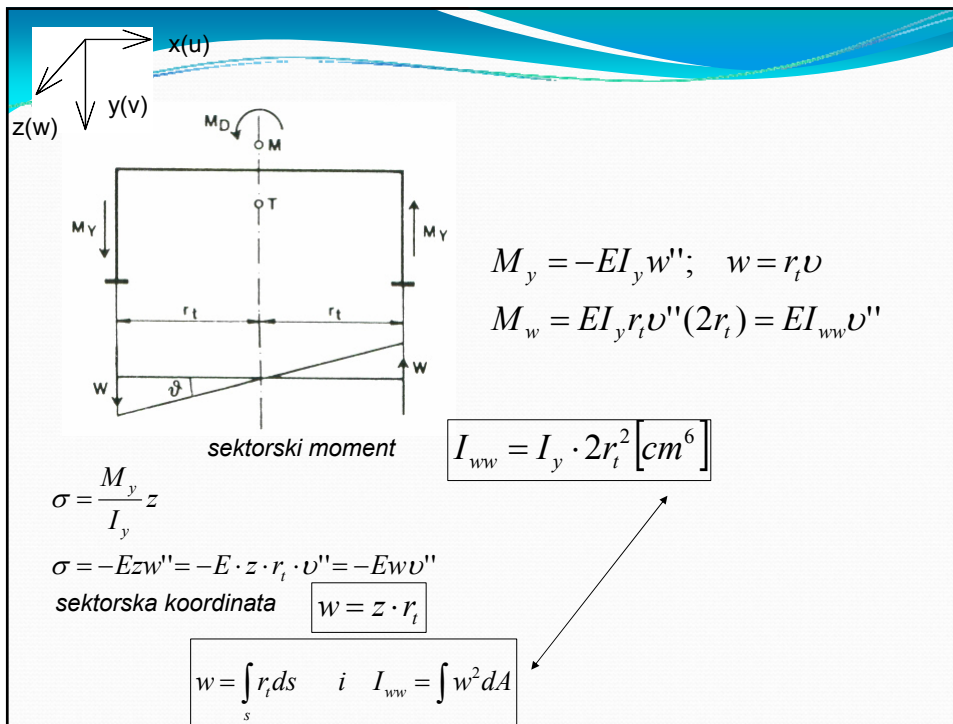
$$\frac{M_D l}{4} \text{ ili } \frac{m_D l^2}{8}$$

TORZIJA I SAVIJANJE



$M_D = Pe \rightarrow$ par momenata savijanja ili bimoment krivljenja

$$M_w = M_y \cdot 2r_t$$



$M_y = -EI_y w''$; $w = r_t v$
 $M_w = EI_y r_t v'' (2r_t) = EI_{ww} v''$

$I_{ww} = I_y \cdot 2r_t^2 [cm^6]$

$\sigma = \frac{M_y}{I_y} z$
 $\sigma = -Ezw'' = -E \cdot z \cdot r_t \cdot v'' = -Ewv''$
 sektorska koordinata $w = z \cdot r_t$

$w = \int_s r_t ds$ i $I_{ww} = \int w^2 dA$

sektorski moment

$\sigma = \frac{M_y}{I_y} \cdot z = \frac{M_w}{I_{ww}} w$

$M_w \dots$ iz kuta zakretanja v
 $M_y \dots$ iz uvjeta ravnoteže

korekcija sektorskog momenta tromosti za zatvoreni presjek

$w = \int_s (r_t - \frac{\psi}{t}) ds$

PODSJETNIK

BREDT 1 $T = \frac{M_T}{2A_m}$ ($M_T = T \oint r_t ds$) $\Rightarrow \tau = \frac{T}{t} = \frac{M_T}{2A_m} \cdot \frac{1}{t}$

BREDT 2 $M_T = GI_T v' \Rightarrow \tau = \frac{GI_T v'}{2A_m t} = G \frac{\psi}{t} \cdot v'$

$\frac{\psi}{t} \dots$ torzijska funkcija $= \frac{I_T}{2A_m t} = \frac{2A_m}{\oint \frac{ds}{t}}$

$$\tau = G \frac{\psi}{t} v' = G \frac{\psi}{t} \frac{M_T}{GI_T} = \frac{M_T}{I_T} \cdot \frac{\psi}{t}$$

analogija s $\sigma = \frac{M_y}{I_s} y$

$$M_T = GI_T v' = \int \tau \left(\frac{\psi}{t} \right) dA = \int G \left(\frac{\psi}{t} \right) v' \left(\frac{\psi}{t} \right) dA = G v' \int \left(\frac{\psi}{t} \right)^2 dA$$

$$\Rightarrow I_T = \int \left(\frac{\psi}{t} \right)^2 dA \quad \dots \quad I_s = \int y^2 dA$$

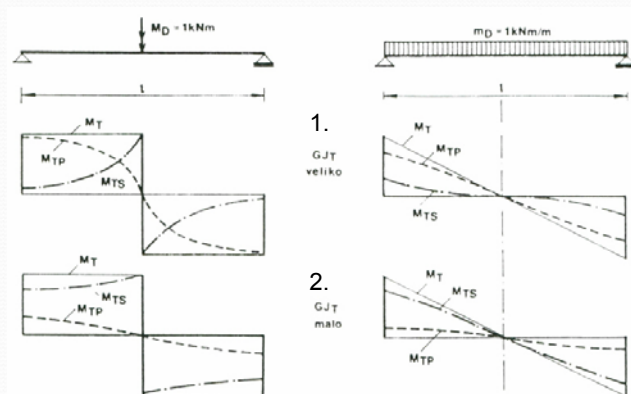
OGRAĐENA TORZIJA

M_D i m_D izazivaju vanjski bimoment M_M $M_M = GI_T v - EI_{ww} v''$

Posmični utjecaj: $\frac{dM_M}{dx} = GI_T v' - EI_{ww} v'''$

ukupna torzija $M_T = M_{TP} + M_{TS}$
 $\frac{(GI_T)_1}{(GI_T)_2} \approx 30$ St. Venant sekundarni

Primjer: $l=70m$



parametar

$$\chi = l \sqrt{\frac{GI_T}{EI_{ww}}}$$

$\chi < 0,5$ vrlo mekano na torziju, mehanizam čistog savijanja

$2 > \chi > 0,5$ miješana torzija, dominantno savijanje

$5 > \chi > 2$ tipično područje miješane torzije

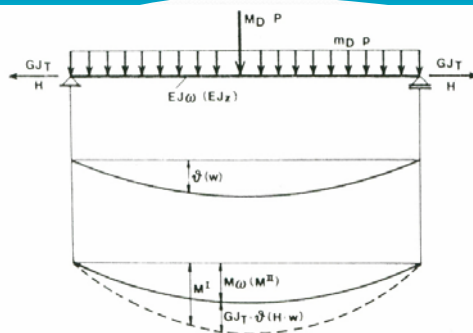
$15 > \chi > 5$ dominira St. Venant, kompaktni sandučasti

$\chi > 15$ čisti St. Venant, krivljenja nema

Napomena: Osjetljivost na oblik presjeka – dijafragme

Aproksimacije

ANALOGIJA SAVIJANJA I TORZIJE



$$EIw'' - Hw = -M_0$$

$$EI_{ww}v'' - GI_Tv = -M_M$$

M^I – teorija 1. reda

M^{II} – teorija 2. reda

$$EIw^{IV} - Hw'' = p_z \quad \text{analogija}$$

$$w \equiv v \quad H \equiv GI_T$$

$$P = M_D \quad p \equiv m_D$$

$$M_y = \int Q_y dx = -EI_y w''$$

$$Q_y = -EI_y w''' = M_y'$$

$$\sigma = \frac{M_y}{I_y} z$$

$$\tau = \frac{QS_y}{I_y t}$$

$$EI_{ww}v^{IV} - GI_Tv'' = m_D$$

$$EI_y \equiv EI_{ww}$$

$$M_w = \int M_{TS} dx = -EI_{ww}v''$$

$$M_{TS} = -EI_{ww}v''' = M_w'$$

$$\sigma_w = \frac{M_w}{I_{ww}} w$$

$$\tau_w = \frac{M_w' \cdot A_w(s)}{I_{ww} \cdot t}$$

savijanje	torzija
diferencijalne jednačbe	
$EJ_y w^{IV} - H \cdot w'' = p_z$	$EJ_{\text{rot}} \vartheta^{IV} - GJ_T \vartheta'' = m_D$
progib w	zakretanje ϑ
vlačna sila H	torzijska krutost GJ_T
krutost protiv savijanja EJ_y	krutost protiv krivljenja EJ_{rot}
koncentrirani teret P	koncentrirani moment M_D
kontinuirani teret p	kontinuirani moment m_D
ležajni uvjeti	
nepokretni ležaj	viljuškasti ležaj
$w = 0$	$\vartheta = 0$
$w'' = 0 (M_y = 0)$	$\vartheta'' = 0 (M_{\text{rot}} = 0)$
upeti ležaj	spriječeno uvijanje
$w = 0$	$\vartheta = 0$
$w' = 0$	$\vartheta' = 0$
slobodni kraj	slobodni kraj
$w'' = 0 (M_y = 0)$	$\vartheta'' = 0 (M_{\text{rot}} = 0)$
moment savijanja	bimoment krivljenja
$M_y = \int Q_y \cdot dx = -EJ_y w''$	$M_{\text{rot}} = \int M_{TS} dx = -EJ_{\text{rot}} \vartheta''$
poprečna sila	posmik bimomenta krivljenja
$Q_y = -EJ_y w''' = M_y'$	$M_{TS} = -EJ_{\text{rot}} \vartheta''' = M_{\text{rot}}'$
naponi	naponi
$\sigma = \frac{M_y}{J_y} \cdot z$	$\sigma_{\text{rot}} = \frac{M_{\text{rot}}}{J_{\text{rot}}} \cdot \omega$
$\tau = -\frac{Q_y \cdot S_y}{J_y \cdot t}$	$\tau_{\text{rot}} = -\frac{M_{\text{rot}}' \cdot A_{\text{rot}}(s)}{J_{\text{rot}} \cdot t}$

Zatvoreni presjeci - korekcija

$$w = \int_s \left(r_t - \frac{\psi}{t} \right) ds$$

Naprezanja

$$M_w \rightarrow \sigma = \frac{M_w}{I_{ww}} w = \frac{M_y}{I_y} \cdot z \left(\frac{2r_t \cdot r_t}{r_t^2} \right) \rightarrow 1$$

Posmik (sekundarni)

$$\begin{cases} M_{TS} = \frac{dM_w}{dx} = -EI_{ww} \psi''' \\ \tau_{TS} = \frac{M_{TS}}{I_{ww}} \cdot \frac{A_w}{t} \end{cases} \quad \text{gdje je} \quad A_w = \int_A w dA$$

sektorski statički moment

→ **sekundarni posmik općenito**

Uz mali GJ_T otvorenih presjeka ovaj posmik odgovara utjecaju poprečne sile od nesimetričnog opterećenja.

Aproksimacija za teoriju 2. reda

$$w = w_0 \frac{1}{1+q} \rightarrow q = \frac{Hw_0}{M_0}$$

$$\text{za } M_0 \rightarrow \text{afino } w_0 \rightarrow M_y = M_0 \cdot \frac{1}{1+q}$$

Primjer sa slike:

$$M_0 = \frac{m_0 \cdot l^2}{8} + \frac{M_0 l}{4}$$

$$w_0 = \frac{5}{384} \frac{m_0 l^4}{EI_{ww}} + \frac{M_D l^3}{48EI_{ww}}$$

$$Hw_0 = GI_T \cdot w_0 \rightarrow q = \frac{Hw_0}{M_0}$$

$$\boxed{M_w = M_0 \frac{1}{1+q}} \rightarrow \sigma_w = \frac{M_w}{I_{ww}} w$$

Posmična komponenta

$$M_{TP} = GI_T \psi'$$

$$\tau_{TP} = \frac{M_{TP}}{I_T} \cdot t_i - \text{otvoreni}$$

$$\tau_{TP} = \frac{M_{TP}}{I_T} \left(\frac{\psi}{t} \right)$$

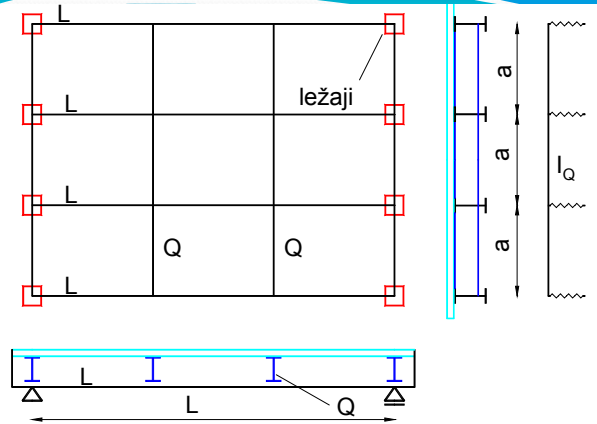
$$\tau_{TP} = \frac{M_{TP}}{2A_m} \frac{1}{t}$$

zatvoreni

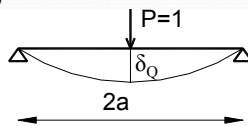
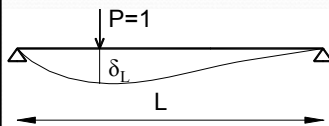
$$M_{TS} = EI_{ww} \psi''' = M_w'$$

$$\tau_{TS} = \frac{M_{TS} A_w}{I_{ww} \cdot t}$$

ROŠTILJNE KONSTRUKCIJE

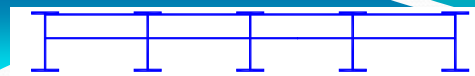


Roštiljna krutost $z = \frac{\delta_L}{\delta_Q}$

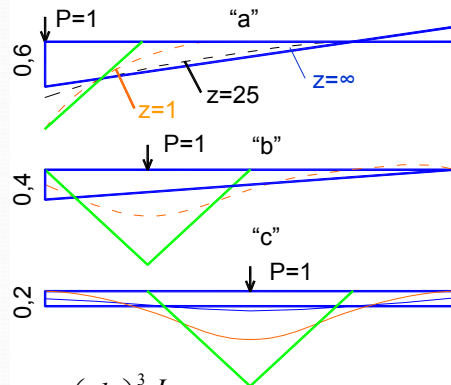


$$\delta_Q = \frac{1 \cdot (2a)^3}{48EI_Q} = \frac{a^3}{6EI_Q} \rightarrow z = \frac{6EI_Q}{a^3} \delta_L$$

$$za \quad L/2 \rightarrow z = \left(\frac{L}{2a}\right)^3 \cdot \frac{I_Q}{I_L}$$



5 GN
1 PN



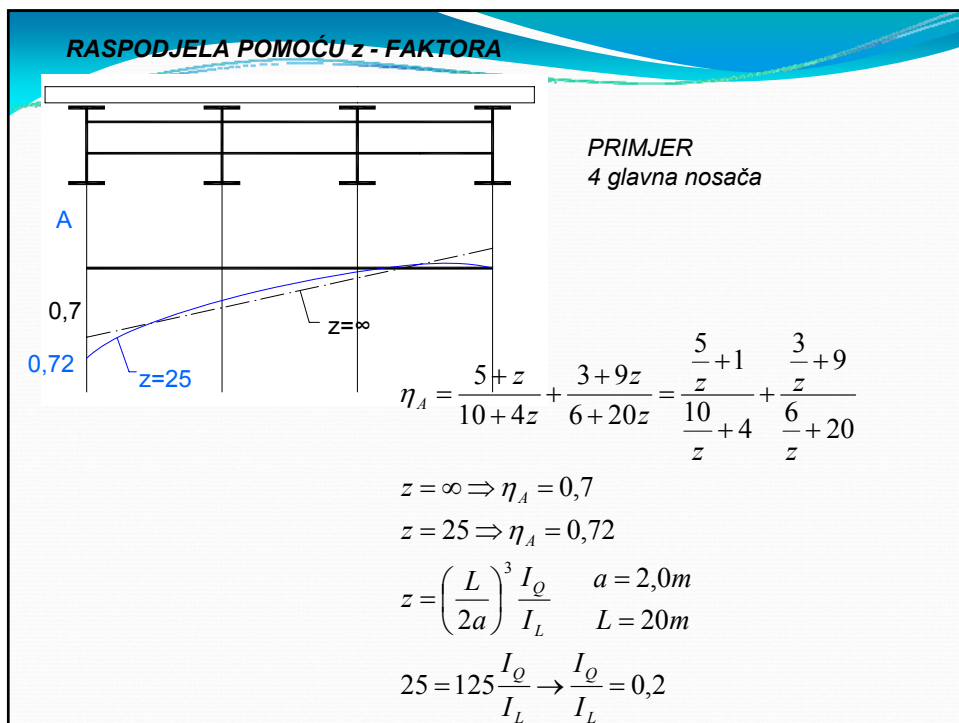
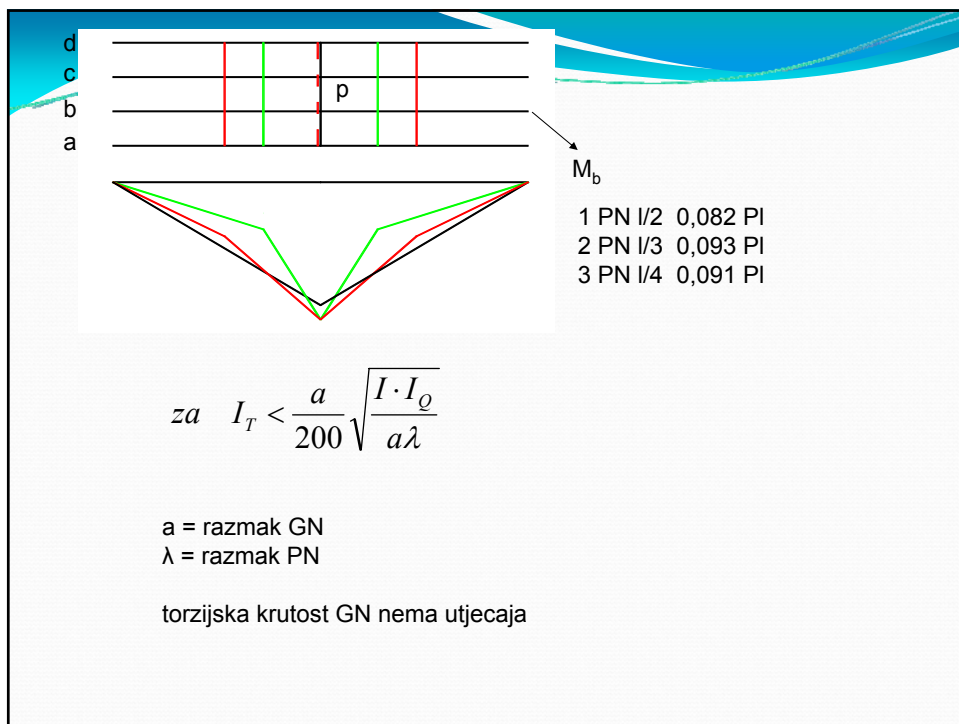
$$z = \left(\frac{l}{2a}\right)^3 \frac{I_Q}{I}$$

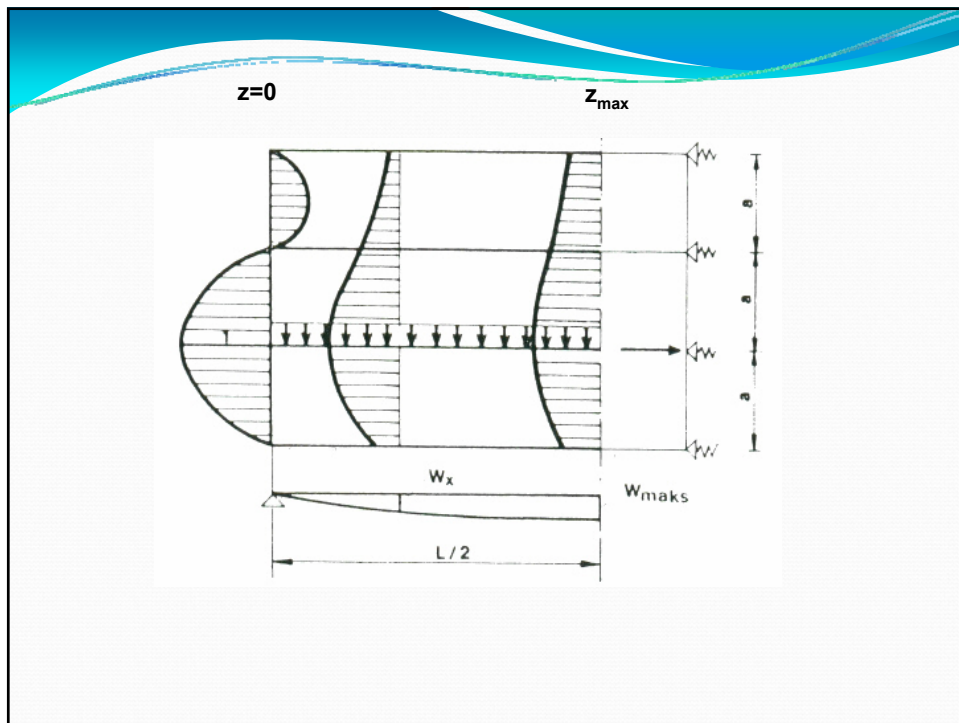
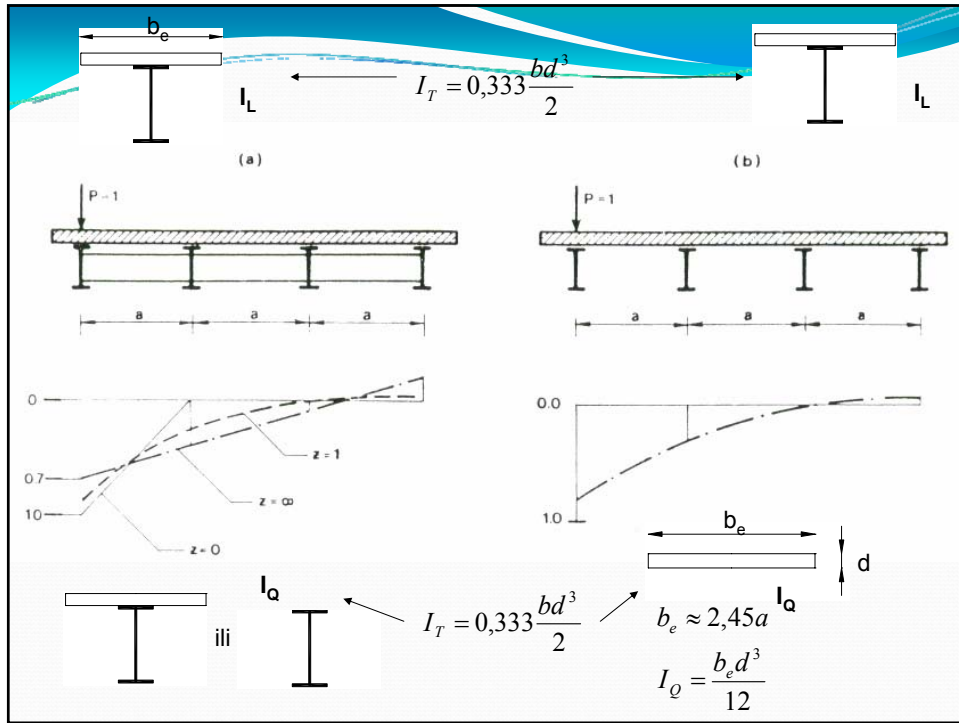
Primjer: $l=20\text{m}$ $a=2,0\text{m}$

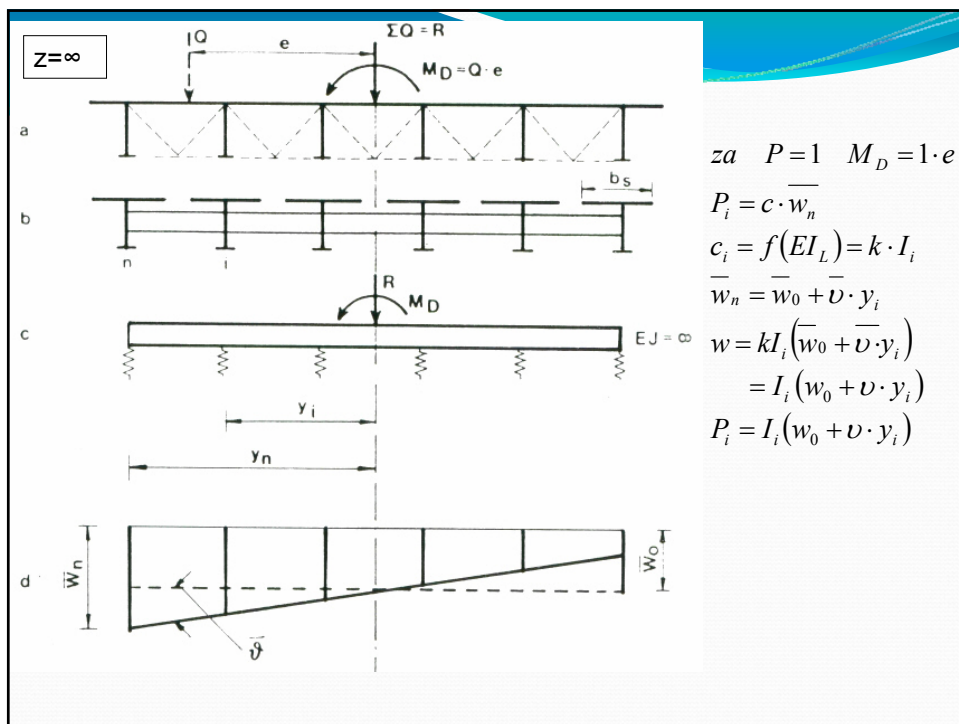
$$z = 1 \Rightarrow I_Q / I = 1/125$$

$$z = 25 \Rightarrow I_Q / I = 1/5$$

$$z_T = \frac{l}{8a} \cdot \frac{EI_Q}{GI_T}$$







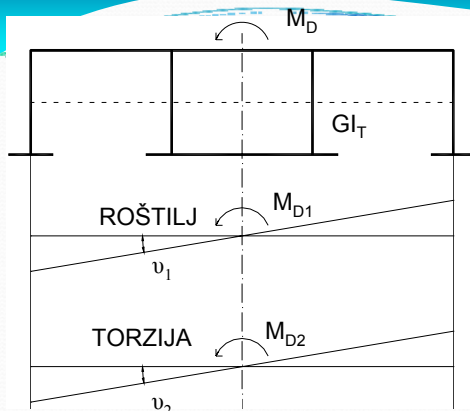
$$P_i = \frac{I_i}{\sum I_i} R + \frac{y_i I_i}{\sum y_i^2 I_i} \cdot M_D$$

za $R = 1$ i $M_D = 1 \cdot e \rightarrow P_i = \eta_i$

$$\eta_i = \frac{I_i}{\sum I_i} + \frac{y_i I_i}{\sum y_i^2 I_i} e = \frac{1}{n} + \frac{y_i e}{\sum y_i^2}$$

$EI_Q \approx \infty$ vrijedi za $L/B > 2$ odnosno $B < L/2$

KOMBINACIJA ROŠTILJA I TORZIJSKE OTPORNOSTI

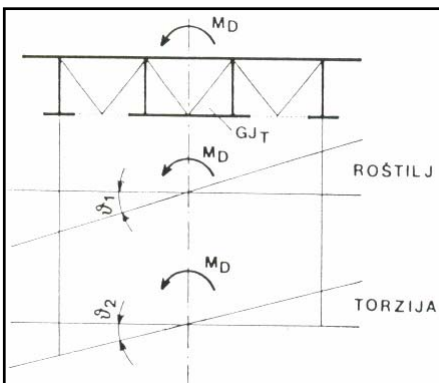


$$\frac{M_{D1}}{M_D} = \frac{\frac{1}{v_1}}{\frac{1}{v_1} + \frac{1}{v_2}} = \frac{v_2}{v_1 + v_2} \rightarrow M_{D1} = \frac{1}{1 + \frac{v_1}{v_2}} \cdot M_D$$

$$\frac{M_{D2}}{M_D} = \frac{v_1}{v_1 + v_2} \rightarrow M_{D2} = \frac{1}{1 + \frac{v_2}{v_1}} \cdot M_D$$

$$v_1 = \frac{\delta_i}{y_i} = \frac{(\overline{P}_i)^3}{y_i \cdot 48EI_i} = \frac{e}{\sum y_i^2 I_i} \cdot \frac{L^3}{48E}; \quad v_2 = \frac{1 \cdot eL}{4GI_T}$$

$$\frac{v_1}{v_2} = \frac{I_T}{\sum y_i^2 I_i} \cdot \frac{L^2}{12} \cdot \frac{G}{E}; \quad (\overline{P}_i) = \frac{y_i e}{\sum y_i^2}$$



$$\delta_i = \frac{y_i J_i \cdot e}{\sum y_i^2 J_i} \cdot \frac{L^3}{48EJ_i}$$

Roštiljno će zakretanje biti:

$$\vartheta_1 = \frac{\delta_i}{y_i} = \frac{e}{\sum y_i^2 J_i} \cdot \frac{L^3}{48E}$$

Torzijsko zakretanje u istom presjeku iz analogije s lančanicom:

$$\vartheta_2 = \frac{1 \cdot e \cdot L}{4 \cdot G \cdot J_T}$$

Prema tome će odnos između ϑ_1 i ϑ_2 biti

$$\frac{\vartheta_1}{\vartheta_2} = \frac{J_T}{\sum_{i=1}^n y_i^2 J_i} \cdot \frac{L^2}{12} \cdot \frac{G}{E}$$

Numerički primjer. Za dispoziciju iz sl. 4.28 zadane su dimenzije: visina nosača $h = 1,0$ m, debljina hrptova $t = 1$ cm, razmak između nosača $a = 4,5$ m, ekvivalentna debljina lima gornjeg pojasa $t = 2$ cm, ekvivalentna debljina donjeg pojasa u srednjoj ćeliji koja zamjenjuje spreg iznosi $t = 0,5$ cm. Raspon konstrukcije $L = 20$ m. Podaci o krutosti: $J_i = 102,2 \cdot 10^{-4} \text{ m}^4$, $J_T = 611,32 \cdot 10^{-4} \text{ m}^4$.

$$G/E = 0,3857, \quad n = 4, \quad \sum_i y_i^2 = 5 \cdot 4,5^2 = 101,25, \quad J_T/J_i = 5,982, \quad L^2/12 = 33,33$$

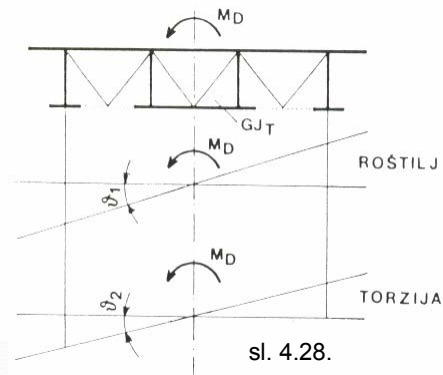
$$\frac{\vartheta_1}{\vartheta_2} = 5,982 \cdot 33,33 \cdot 0,3857 \cdot 1 / 101,25 = 0,760$$

Korekcija roštiljne komponente:

$$K_R = \frac{1}{1 + 0,760} = 0,568.$$

Torzijska komponenta:

$$K_T = \frac{0,760}{1 + 0,760} = 0,432.$$



sl. 4.28.

Poprečna roštiljna raspodjela za krajnji nosač uz $y_i = 6,75$ i $M_D = 1 \cdot 6,75$. Prema izrazu 4.53a dobivamo:

$$\eta_{iR} = \frac{1}{4} + \frac{6,75}{5 \cdot 4,5^2} \cdot 0,568 \cdot 6,75 = 0,25 + 0,26 = 0,51 < 0,70$$

(usporedi sl. 4.25. za $z = \infty$)

Torzijom preko posmičnog toka prenosi se:

$$\eta_{iT} = 0,432 \cdot 6,75 = 2,92 \text{ (m)}$$

U konkretnom primjeru iskorišten je donji vjetrovni spreg između dvaju srednjih uzdužnih nosača preko kojeg se može prenijeti 43% nesimetričnog opterećenja, tako da je roštiljna komponenta bitno smanjena.

OPTIMALNE VISINE PUNOSTIJIENIH GLAVNIH NOSAČA

- utrošak materijala za cestovne mostove po jedinici korisne površine variraju 40 do 70% u odnosu na prosjek, a za željezničke 15 do 30%
- odstupanja se mogu pripisati različitim normiranim uvjetima u raznim zemljama Europe ali i nedostatku potrebnih analiza i preporuka
- analize optimalnog rasporeda materijala za uskopojasne čelične nosače željezničkih mostova obavljene su prije 50ak godina i mogu se i danas zadovoljavajuće koristiti
- prema tim analizama minimalni potrošak materijala kod: $h = L/10$ do $L/16$
- detaljna analiza – granično stanje nosivosti i uporabno granično stanje

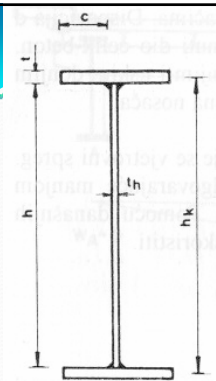
dopušteni napon:

$$\gamma \cdot M \leq M_u \quad M_u = W_{pot} \cdot \sigma_y \quad W_{pot} = \gamma \cdot M / \sigma_y$$

dopušteni progib:

$$f_p / L \leq 1 / K$$

f_p – progib od pokretnog opt. bez dinamičkog faktora
 L – raspon nosača
 K – iskustvena veličina 500-1000
 500: cestovni mostovi
 700-800: željeznički mostovi
 1000: rešetkasti glavni nosači



PRIMJER DETALJNE ANALIZE:

$$A_c = 2 \cdot A_p + t \cdot h$$

$$I = t \cdot h^3 / 12 + 2 \cdot A_p \cdot h^2 / 4 = (h^2 / 4) (A_c - t \cdot h \cdot 2 / 3)$$

$$W = 2 \cdot I / h = (h / 2) (A_c - t \cdot h \cdot 2 / 3)$$

$$A_c = 4 \cdot I / h^2 + t \cdot h \cdot 2 / 3$$

$$A_c = 2 \cdot W / h + t \cdot h \cdot 2 / 3$$

potreban moment tromosti $I = I_{pot}$ dobijemo iz:

$$f_p / L \leq 1 / K; \quad 5 \cdot M_p \cdot L / 48 \cdot EI \leq 1 / K \quad \text{i odavde}$$

$$I_{pot} \geq 5 \cdot M_p \cdot L \cdot K / 48 \cdot E$$

jednadžba potroška materijala:

$$A_c = (20 \cdot M_p \cdot L \cdot K / h^2 \cdot 48 \cdot E) + (t \cdot h \cdot 2 / 3)$$

potreban moment otpora $W = W_{pot}$ dobijemo iz:

$$W_{pot} = M / \sigma_d \quad \text{ili} \quad W_{pot} = \gamma \cdot M / \sigma_y$$

jednadžba potroška materijala iz uvjeta nosivosti:

$$A_c = 2 \cdot M \cdot \gamma / h \cdot \sigma_y + t \cdot h \cdot 2 / 3$$

Obje funkcije potroška materijala imaju svoj minimum koji ovisi o konstruktivnoj visini h i pronalaženje tog minimuma daje optimalnu konstruktivnu visinu.

Ograničenja uzrokovana progibanjem su danas ublažena pa u analizi ostaje samo za riješiti minimum iz uvjeta nosivosti:

$$\frac{dA_c}{dh} = 0; \quad -2M\gamma / h^2 \sigma_y + \frac{2}{3}t = 0 \rightarrow h = \sqrt{\frac{3M\gamma}{t\sigma_y}}$$

(M promjenjiv $\Rightarrow h$ promjenjiv)

Dodatan čelik je potreban za stabilizaciju, spregove i spojeve.

OPTIMALNE VISINE ZA:

• SLOBODNO POLOŽENE GREDE

a) opće variranje presjeka

$$t = \text{const.} \quad h = \sqrt{\frac{6}{1+\alpha} \frac{W_u}{C \cdot t}}$$

$$C = 2,0 - 2,3$$

b) variranje samo hrpta i donjeg pojasa
dok je gornji pojas konstantan

$$t = \text{const.} \quad h = \sqrt{\frac{W_u}{C \cdot t}}$$

$$C = 2,7 - 3,3$$

• KONTINUIRANE NOSAČE

a) opće variranje presjeka

$$t = \text{const.} \quad h = \sqrt{\frac{6W_u}{C_k \cdot t}}$$

$$C_k = 2,8 - 3,1$$

b) variranje samo hrpta i donjeg pojasa

$$t = \text{const.} \quad h = \sqrt{\frac{2W_u}{C_k \cdot t}}$$

$$C = 2,3 - 2,7$$

$$\alpha = \frac{\sigma_o}{\sigma_u}; \quad W_u = \frac{\gamma \cdot M_{\max}}{\sigma_y}$$

t – debljina hrpta
 σ_o – granični napon u gornjem pojasu
 σ_u – granični napon u donjem pojasu
 C – konstrukcijski faktor koji uzima u obzir promjenjivi presjek pojasa, stupanj iskorištenosti i potrebni dodatni materijal

- odstupanja od optimalne visine povećavaju potrošak materijala:

- 20-30% veća visina => 3-8% više materijala
- 30% manja visina => 6-12% više materijala

- maksimalna odstupanja zbog pogreške procjene konstrukcijskog faktora mogu smanjiti ili povećati visine od 10 do 15%. Za takva odstupanja potrošak materijala može se povećati najviše 2%

- primjer:
 - britanske preporuke za cestovne mostove – optimalna visina $L/20$

OPTIMALNE VISINE

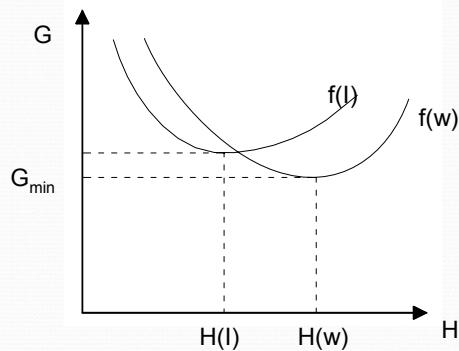
Utrošak materijala = $f(h)$

$f(l)$

$f(w)$

Statistika - za željezničke v~15-30%
- za cestovne v~40-70%

Osnovni parametar utroška je H .



$$G = C \cdot \gamma \cdot A_c \cdot l$$

C — konst. faktor
 γ — spec. masa
 A_c — površina
 l —

za I presjek:

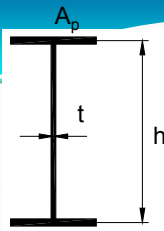
$$f(I) = A_c = \frac{4I}{H^2} + \frac{2}{3}tH$$

$$f(w) = A_c = \frac{2W}{H} + \frac{2}{3}tH$$

$$I = I_{pot} \quad \text{iz uvjeta progibanja}$$

$$W = W_{pot} = \frac{\gamma M}{\sigma_u} \quad \text{iz uvjeta nosivosti}$$

$$\frac{dA_c}{dM} = 0 \rightarrow H = f \begin{cases} I_{pot} \\ W_{pot} \end{cases}$$



$$I = \frac{th^3}{12} + 2A_p \frac{h^2}{4} = \frac{h^2}{4} \left(A_c - \frac{2}{3}th \right)$$

$$A_c = th + 2A_p$$

$$w = \frac{2I}{h} = \frac{h}{2} \left(A_c - \frac{2}{3}th \right)$$

$$A_c = \frac{4I}{h^2} + \frac{2}{3}th$$

$$A_c = \frac{2W}{h} + \frac{2}{3}th$$

$$I = I_{pot} \quad \dots \quad \text{iz} \quad \frac{f_p}{L} \leq \frac{1}{K} \quad (K = 500, 700, 1000)$$

$$W = W_{pot} = \frac{M_{\max}}{\sigma_{doz}} = \dots = \frac{\gamma M}{\sigma_v}$$

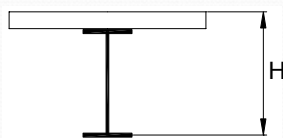
$$\frac{\partial A_c}{\partial h} = 0 = -\frac{2W}{h^2} + \frac{2}{3}t \rightarrow h = \sqrt{\frac{3M\gamma}{t\sigma_v}}$$

Grube aproksimacije

1) Željeznički uskopojasni GN:

$$H_{opt} = \frac{L}{10} - \frac{L}{16}$$

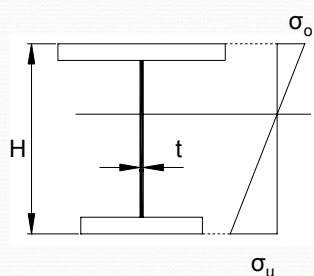
2) Cestovni širokopojasni:



$$H \approx \frac{L}{15} - \frac{L}{20}$$

za $L=20-30\text{m}$ $L=30-50\text{m}$

Točnije vrijednosti u funkciji w_{pot}



$$\alpha = \frac{\sigma_o}{\sigma_u}; \quad W_o = \frac{M_{max}}{\sigma_o}$$

$$W_u = \frac{M_{max}}{\sigma_u}$$

$$H = \sqrt{\frac{6}{1+\alpha} \cdot \frac{W_u}{C \cdot t}}$$

$C = 2 - 2,3$ za položene grede

$C = 2,8 - 3,1$ za kontinuirane grede